## NIELS BOHR's

## contribution to

## **EPISTEMOLOGY**

By L. Rosenfeld

HEN I was staying at Yukawa's Institute in Kyoto two years ago, I had occasion to discuss Bohr's ideas with the great Japanese physicist, whose conception of the meson with its complementary aspects of elementary particle and field of nuclear force is one of the most striking illustrations of the fruitfulness of the new way of looking at things that we owe to Niels Bohr. I asked Yukawa whether the Japanese physicists had experienced the same difficulty as their Western colleagues in assimilating the idea of complementarity and in adapting themselves to it. He answered, "No, Bohr's argumentation has always appeared quite evident to us;" and, as I expressed surprise, he added, with his aristocratic smile, "You see, we in Japan have not been corrupted by Aristotle."

If Yukawa had also mentioned Plato, his epigram would have given a complete characterization, which it would be difficult to make more pregnant, of the significance of Bohr's contribution to philosophical thought. Untrammelled by formal schooling, guided only by the sure intuition of the investigator of nature, Bohr rediscovered the dialectical process of cognition which had so long been obscured by the unilateral development of epistemology on the basis of Aristotelian logic and Platonic idealism. Indeed, when Newton formulated the new natural philosophy (as he called it) which was to give modern science its aim and its method, he was so scared at the prospect of the impious material-

ism to which exclusive reliance on human reason would lead that he injected into his philosophy a generous dose of mystical antidote of Platonic inspiration; and this uneasy alliance of rationalism and mysticism has since then been paralyzing scientific philosophy. Even those scientists of the nineteenth century who were most inclined to idealism were impervious (as the example of Oersted shows) to the Hegelian form of dialectics; as for the Marxist dialectics, which might have given the materialists among them a clue, it was effectively hidden from their view by an impenetrable social barrier.

When young Niels Bohr entered the scene, an antimaterialistic reaction was at its height. His father, an eminent physiologist, firmly rejected the superficial materialism of the Haeckel school; and this condemnation appeared the more significant as it came from a master in the quantitative investigation of the physical and chemical processes underlying the physiological functions. Just because of this first-hand experience, Bohr's father was in a good position to know how sterile materialistic speculations remained for the analysis of concrete cases, and to appreciate the indispensable part played in such analysis by specific biological concepts, such as that of function. Hence his insistence on the necessity of upholding, at least for methodical reasons, teleological viewpoints in biology.

This argumentation of his father, solidly founded on the conditions of actual scientific investigation, impressed Bohr much more than any teaching of traditional philosophy. The philosopher Høffding, a wise and broad-minded thinker, belonged to the

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circle of the father's nearest friends, and soon formed a trusting affection for the earnest and sensitive adolescent; but his influence on Bohr was more marked in the general attitude to human problems than in questions of epistemology. Although Høffding showed more appreciation than was common at the time of the role of science in the development of philosophical conceptions, he lacked the intimate knowledge of scientific thinking necessary for a detailed analysis of its philosophical implications. A little-known incident illustrates at the same time Hoffding's shortcomings in the more technical aspects of philosophical analysis and the seriousness with which Bohr applied himself to their study: When he attended Hoffding's course on formal logic, Bohr noticed some serious errors in the professor's exposition, which, with his wonted candor, he pointed out to him. Høffding took the student's censure quite philosophically, and he even submitted the proofs of the amended edition of his course to his young friend's approval. In later life, Bohr remembered the fact, but had forgotten what the errors in question actually were.

Høffding's lectures on the history of philosophy had, at any rate, the merit of not enforcing any system upon his audience. He was more concerned to present the problems than the solutions proposed by the various systems; for, he said, the systems come and go, but the problems remain. It was not, however, in Bohr's nature to be attracted by anything so vague as the classical problems of philosophy; his interest could only be awakened by some concrete, sharply outlined situation which would grip his imagination. He liked Spinoza's conception of the psychophysical parallelism, in which he later was tempted to see an endeavor to express the complementarity relations in the realm of psychological phenomena; eventually, however, when his conception of complementarity had reached full maturity, he abandoned this interpretation, and rejected the whole idea of a psychophysical parallelism in the spirit of Spinoza, as implying a kind of mutual determination and equivalence of the two parallel aspects which certainly does not belong to a true relation of complementarity. He read Kierkegaard, but admired the virtuosity of his style more than his highly strung meditations. What made a really deep and lasting impression on him, however, was the unpretentious "Tale of a Danish Student", in which Poul Martin Møller has given such a delightfully humorous illustration of Hegelian dialectics.

Here at least, in the character of a soul-searching licentiate desperately struggling to keep his ideas together, he could see a vivid picture of the dramatic situation which might confront the unwary ponderer on the intricacies of human thinking: 80

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"... and then I come to think of my thinking about it; again I think that I think of my thinking about it, and divide myself into an infinitely retreating succession of egoes observing each other. I don't know which ego is the real one to stop at, for as soon as I stop at any one of them, it is another ego again that stops at it. My head gets all in a whirl with dizziness, as if I were peering down a bottomless chasm, and the end of my thinking is a horrible headache."

Habent sua fata libelli! Poul Møller had little imagined that his light-handed banter would one day start a train of thought leading to the elucidation of the most fundamental aspects of atomic theory and the renovation of the philosophy of science. Yet it is hardly an exaggeration to say that the perplexities of this licentiate, especially his struggle with his many egoes, were the only object lesson in dialectical thinking that Bohr ever received, and the only link between his highly original reflection and philosophical tradition.

Let us quote another characteristic scene, in which the licentiate tries to explain to his cousin Fritz why it takes him such a long time to write his dissertation:

". . . certainly I have seen before thoughts put on paper; but since I have come distinctly to perceive the contradiction implied in such an action, I feel completely incapable of forming a single written sentence. And although experience has shown innumerable times that it can be done, I torture myself to solve the unaccountable puzzle, how one can think, talk, or write. You see, my friend, a movement presupposes a direction. The mind cannot proceed without moving along a certain line; but before following this line, it must already have thought it. Therefore one has already thought every thought before one thinks it. Thus every thought, which seems the work of a minute, presupposes an eternity. This could almost drive me to madness. How could then any thought arise, since it must have existed before it is produced? When you write a sentence, you must have it in your head before you write it; but before you have it in your head, you must have thought it, otherwise how could you know that a sentence can be produced? And before you think it, you must have had an idea of it, otherwise how could it have occurred to you to think it? And so it goes on to infinity, and this infinity is enclosed in an instant.

"Bless me," said Fritz with indifference, "while you are proving that thoughts cannot move, yours are proceeding briskly forth!"

"That is just the knot," replied the licentiate, "This increases the hopeless mix-up, which no mortal can ever sort out. The insight into the impossibility of thinking contains itself an impossibility, the recognition of which again implies an inexplicable contradiction."

Bohr must have recalled this passage as he manifested such amusement when Dirac, seeing him hesitate in the formulation of some remark, observed sententiously, "One should never start a sentence before knowing the end of it." Could it be that one reason why Bohr took such a fancy to the poor licentiate's embarrassment in writing a sentence or keeping the subject of his dissertation within bounds was that he saw in him a goodhumored caricature of himself? From the time when we were investigating together the complementarity relations of electromagnetic quantities, I remember that on one of those days when the solution of our problem seemed more elusive than ever, we took a bicycle ride in the country and came in sight of a beautiful mansion, called Nærumgaard, where Bohr's grandparents had lived. "It is here," said Bohr, "that I finished my first paper." (This was the determination of the surface tension of water that he carried out in his student years and that won him a gold medal of the Danish Academy of Sciences.) "I was working on it in Copenhagen, but the experiments had no end; I always noticed new details that I thought I had first to understand. At last my father sent me out here, away from the laboratory, and I had to write up the paper." Now it was just part of Bohr's greatness that he did finish his papers. He might pursue a problem for years on end, with dogged tenacity, looking at it from all angles, taking up the same point over and over again, but he knew when to stop. He vividly realized that our proud theories are but temporary resting places of the mind on the unending road to knowledge: Such resting places, however, there must be, where we may taste the joys of knowledge, feel that we have reached a certain harmony between our mental picture of the world and our experience of it.

It is remarkable that Bohr's first preoccupation with philosophical problems did not arise from his physical investigations, but from general epistemological considerations about the function of language as a means of communicating experience. The fictitious case of the licentiate trying to sort out his egoes was only an exaggeration of more common situations in which the same word is used, in different contexts, to denote aspects of human experience which are not only different, but even mutually exclusive; thus we currently use the same word when we speak of a state of our consciousness and of the concomitant behavior of our body. How to avoid ambiguity in such cases was the problem that worried Bohr. In his search for a solution, he was guided by a beautiful analogy with

the concepts of multiform function and Riemann surface. The ambiguity of every word referring to our mental activity may be expressed by saving that it belongs to different "planes of objectivity" just as the values of a multiform function are distributed on different Riemann planes. The use of words in everyday life must then be subject to the condition that they be kept within the same plane of objectivity; and as soon as we deal with words referring to our own thinking, we are exposed to the danger of gliding onto another plane. In mathematics, that highly sophisticated language, we are guarded from this danger by the essential rule never to refer to ourselves. But just as the gist of Riemann's conception lies in regarding all the branches of a multiform function as one single function, it is an essential feature of ordinary language that there is one word only for the different aspects of a given form of psychical activity. We cannot hope, therefore, to avoid such deep-rooted ambiguities by creating "new concepts". We must rather recognize the mutual relationships of the planes of objectivity as primitive, irreducible ones, and try to remain keenly aware of them.

As to the origin of this fundamental peculiarity of human language, it could be traced directly to man's position in the universe, which conferred on him the unique possibility of making himself the object of his own observation. Here again, we hear the licentiate describe the situation in his own way:

"Thus on many occasions man divides himself into two persons, one of whom tries to fool the other, while a third one, who in fact is the same as the other two, is filled with wonder at this confusion. In short, thinking becomes dramatic and quietly acts the most complicated plots with itself and for itself; and the spectator again and again becomes actor."

Does not this last sentence call forth in our memories familiar echoes of one of Bohr's favorite reflections? Thus we find foreshadowed in these youthful meditations all the main themes that recur, steadily amplified, in his later epistemological work: the use of language for the objective communication of experience; the ensuing necessity of fixing the unambiguous meaning of words by reference to situations of common experience; and finally, the possible occurrence of a duality of aspects requiring special caution in the use of language to secure unambiguous communication.

Against this background it is easier to imagine Bohr's reaction to the dilemma about the dual nature of light, which had puzzled the physicists since Planck's discovery of the quantum of action had led Einstein to emphasize the corpuscular features of light in its interaction with matter, contrasting with the wave-like manner in which it spreads out in space. While the great masters were vainly trying to eliminate the contradiction in Aristotelian fashion by reducing one of the aspects to the other, Bohr realized the futility of such attempts; he knew that we had to live with this dilemma as with the others he had so deeply pondered over, and that the real problem was to refine the language of physics so as to provide room for the coexistence of the two conceptions of light, with suitable precautions in order to avoid any ambiguity in their application. Of course, this was just a program, and many years were to pass before it could be accomplished; but we see that the point of view of complementarity (to call it by its future name) was a constructive one, preventing effort from being wasted in sterile channels and exhibiting much more sharply than the current view the true significance of the quantum of action. There are still too many people who are prone to speak lightly of complementarity as "just philosophy" because they fail to realize its importance as a guiding principle through the whole historical development of quantum theory.

In the decade following Planck's discovery, and especially after Rutherford's establishment of the existence of the atomic nucleus, it had become clear that the quantum of action intervened just as essentially in the constitution of atomic systems as in the nature of light; but there was here also a sharp contradiction between the mechanical properties of such systems and their quantal features. Just in the sharpness of this new contradiction, however, Bohr saw the hope of decisive progress; just because every other way out was barred, he boldly put forward his quantum postulates, feeling absolutely convinced that they were the expression of a fundamental truth. The quiet firmness of his attitude in the face of the general confusion then prevailing is a striking testimony to the strength he derived from his epistemological standpoint.

Most revealing of his dialectical turn of mind at this early stage is the concluding sentence of a lecture he gave in 1913 on his explanation of the Balmer formula; after stressing the contrast between the classical and quantal behavior of atomic systems, he ended with the remarkably prophetic statement: "Just by accentuating this contrast it will perhaps be possible in the course of time to bring about a certain consistency also in the new ideas."

When at last, with the advent of quantum mechanics, the long-sought logical foundation was established for a rigorous formulation of the complementary relationships of atomic theory, Bohr was not content to point out how this new development fulfilled his far-sighted expectations; he eagerly took up the task of refining the complementarity idea by a minute analysis of its multiple implications. In this work, he was to no small extent stimulated by Einstein's criticism; the account of this prolonged controversy, with its dramatic succession of shrewd objections and brilliant refutations, has been given by Bohr himself in a later paper which will remain a classic in the history of physics and in the history of philosophy.

The method to which the two giants had recourse in this memorable contest deserves special mention: in order to visualize more easily the meaning of the concepts at issue, they imagined more or less idealized experimental arrangements illustrating the way in which these concepts are actually applied. In devising such imaginary experiments, Einstein's inventiveness was supreme; but Bohr was unsurpassed for penetrating analysis of their implications. His greatest triumph was the successful refutation of the paradox which will remain part of the physical tradition under the name of the "Einstein box". Einstein had argued that it should be possible, in contradiction to the indeterminacy relation, to ascertain accurately the energy carried off by a pulse of light emitted at a definite time. He imagined a box filled with electromagnetic radiation, from which a pulse of radiation could be emitted by the clock-controlled opening and closing of a diaphragm; one could then measure the emitted amount of energy by weighing the box before and after the operation of the diaphragm. In examining the detailed execution of this experiment, Bohr saw that, for a given accuracy of the weighing of the box, its displacement in the gravitational field utilized for this weighing implied a change in the rate of the clock governing the emission because of a well-known consequence of Einstein's own theory of gravitation. That change was just large enough to spoil our control of the time evolution of the emission process in conformity with expectation from the indeterminacy relation. When he later retold this argument, which he was never tired of doing, Bohr used to add that he did not understand how Einstein could ever imagine that the facts which had found expression in quantum theory would contradict the invariance requirements to which he had given such a beautiful formulation by his principles of general relativity This, however, I always felt was a comment post eventum. I happened to see Bohr on the very eve-

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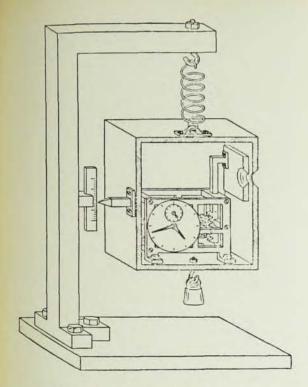
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Bohr's "realistic" representation of the "Einstein box" shows a mock-realistic style of drawing intended to stress the classical nature of the measuring apparatus, as well as the necessity of carefully analyzing the measuring process and not being satisfied with too schematic arguments, which may be treacherous. The formulae

$$\begin{split} \Delta \dot{p} &\approx \frac{h}{\Delta x} < T \cdot g \cdot \Delta m \\ &\frac{\Delta T}{T} = \frac{1}{c^2} g \Delta x \\ &\Delta T > \frac{h}{c^2 \Delta m} \\ &\Delta T \cdot \Delta E > h \end{split}$$

summarize the analysis: in order to obtain an accuracy  $\Delta m$  in the determination of the mass, or  $\Delta E = c^2 \Delta m$  in the determination of the energy, by weighing during a time T in a gravitational field of acceleration g, the accuracy of the determination of the momentum acquired by the box in this weighing must be  $\Delta p$ , implying an uncontrolled displacement  $\Delta x$  of the box. This in turn involves an error  $\Delta T$  in the rate of the clock. Combining the first two relations, one obtains the third, or equivalently the fourth, which is the indeterminacy relation between time and energy. For full details, see N. Bohr, "Discussion with Einstein on epistemological problems in atomic physics", in Albert Einstein, Philosopher-Scientist (Library of Living Philosophers, Evanston, Ill., 1949) p. 201 [reprinted in Niels Bohr, Atomic Physics and Human Knowledge (Wiley, New York, 1958)].

ning after he had just had the paradox proposed to him by Einstein, and I can testify that he was then very far from such lofty considerations.

It is highly instructive to observe how Bohr was driven step by step to a more and more precise recognition of the conditions which had to be specified in order to achieve full harmony between the statements of the theory and the experiments devised to check them. The method reached its utmost refinement in the examination of the fundamental concepts of electromagnetic theory, in which I had the privilege of participating. In this case, one had to exploit to the last limit all possibilities of controlling the course of the measuring process before complete agreement could be reached with the theoretical predictions. Thus, suppose that the spacetime situation of two domains in which measurements of electromagnetic field quantities are performed is such that radiation from each of them is able to reach the other within the time taken by the measurements. Then the product of the measuring errors will not be reduced to the theoretical minimum unless we avail ourselves of the possibility of exchanging information between the two domains and establish between the two sets of test bodies an automatic signalling system enabling them to compensate their mutual perturbations in the most effective manner.

As the outcome of this patient and unrelenting effort of elucidation, there emerged an entirely novel view of the meaning and structure of physical description. Our experiments, to use Bohr's favorite phrase, are questions that we put to Nature; and in our theories we try to state what we have learned from her in a language ensuring unambiguous and objective communication. Hence the paramount importance of establishing a rigorous terminology, sufficiently general to make due allowance for the peculiar conditions under which we observe atomic systems. We let such systems interact with apparatus which is necessarily adapted to our means of immediate perception, and whose behavior is accordingly described in the language of classical physics. In the account of any atomic phenomenon, we must include a specification of the experimental arrangement fixing the conditions of observation, but a sharp distinction must be made between the part of the total system which forms the apparatus and the part which is the object of observations, and which may exhibit quantal properties. Moreover, the complete definition of the phenomenon must essentially contain the indication of some permanent mark left upon a recording device which is part of the apparatus; only by thus envisaging the phenomenon as a closed event, terminated by a permanent record, can we do justice to the typical wholeness of the quantal processes and steer clear of all the paradoxes to which a lax terminology may lead.

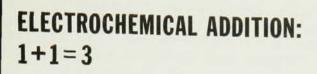
It is in this widened framework that the conception of complementarity in all its generality finds its natural place, as a logical relationship between two physical phenomena, both representing aspects of a physical system equally necessary for its complete description, but corresponding to mutually exclusive experimental conditions. The occurrence of such relationships in atomic physics has a momentous consequence for the causality relations between atomic phenomena: this causality can no longer be deterministic, as it is for large-scale events, but has to be fundamentally statistical. Moreover, the function of a physical theory appears itself in a new light: in view of the necessity of explicitly specifying, for each phenomenon, the conditions of its observation, the description of the phenomena is no longer, as in classical physics, a picture of events from which all reference to observation is eliminated; instead, it appears essentially as a rational and fully objective account of the interaction of the external world with human observers-a conception of science which obviously conforms much better to the part it actually plays in human society.

It is now thirty years since the clarification of the complementary features of quantum mechanics and quantum electrodynamics was essentially completed. On the face of it, physicists at large have reconciled themselves with this intrusion of dialectics into their traditional modes of thought. Who among us, however, could boast of having mastered all the intricacies of complementarity arguments to the point of being prepared for any situation? I would rather characterize our attitude in the words that I once heard from one of Bohr's most faithful and eminent disciples, Weisskopf, who, after having discovered, not without effort, the explanation of a tricky case of complementarity put to him by a doubting experimenter, exclaimed, "Bohr always wins!" Too few even now realize the earnestness of the epistemological issues with which Bohr had to contend single-handed; too few fathom the depth of the problems he had solved and appreciate the revolutionary significance of his contribution to a better understanding of the dialectic process and the development of a truly scientific philosophy.

Complementarity is no system, no doctrine with ready-made precepts. There is no via regia to it; no formal definition of it can even be found in Bohr's writings, and this worries many people. The French are shocked by this breach of the Cartesian rules: they blame Bohr for indulging in "clair-obscur" and shrouding himself in "les brumes du Nord". The Germans in their thoroughness have been at work distinguishing several forms of complementarity and studying, in hundreds of pages, their relations to Kant. Pragmatic Americans have dissected complementarity with the scalpel of symbolic logic and undertaken to define this gentle art of the correct use of words without using any words at all. Bohr was content to teach by example. He often evoked the thinkers of the past who had intuitively recognized dialectical aspects of existence and endeavored to give them poetical or philosophical expression; our only advantage over these great men, he would observe, is that in physics we have been presented with such a simple and clear case of complementarity that we are able to study it in detail and thus arrive at a precise formulation of a logical relationship of universal scope. The nature of this relation he regarded as sufficiently illustrated by his analyses of the limits of validity of classical physical concepts.

On the numerous occasions on which he addressed assemblies of specialists in the most varied disciplines, Bohr never failed to acquaint them with the idea of complementarity as it had taken shape in atomic physics, and to suggest possible applications in the realm of their own investigations. Psychical phenomena of course furnish striking examples of complementarity. In fact, in pointing to such examples, Bohr was returning to the very source of inspiration for his earliest dialectic meditations. He was always as eager to learn as to teach, however, and when much later, some time in the early thirties, his old friend the psychologist Rubin called his attention to William James' Principles, he joyfully recognized in this great book a general attitude akin to his own; he was particularly enthusiastic about James' brilliant description of the stream of consciousness. He also gave much thought to the complementary aspects of human relations. Above all he was concerned to point out that, in ethical and social questions the recognition of complementarity between seemingly irreconcilable points of view could help people to get rid of prejudices fostering intolerance; but the way in which hints of such prospects were generally received by church dignitaries and other pillars of society suggests that these problems are not yet ripe for an approach on so rational lines.

The complementarity relations in biology were constantly in Bohr's mind; it was a matter of par-



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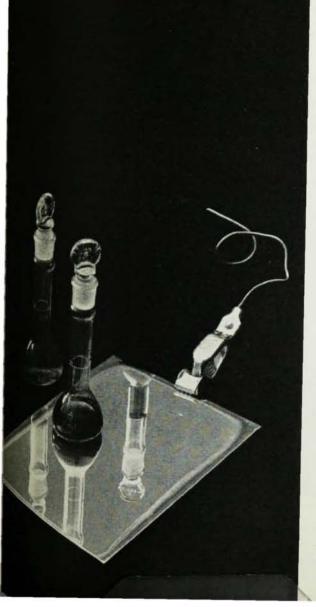
This one-plus-one-equals-three synergistic effect has been widely used in decorative plating. Now, supported by unique autoradiographic evidence, we have postulated a theoretical explanation. Details are available on request. Briefly, it involves ion interactions and adsorptions and gets deeper into matters of ion and atom bonding and bond breaking—subjects typical of persistent basic research in General Motors

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	Before plating	With A alone	With B alone	With A & B
Amount of A adsorbed	0	113	0	189
Amount of B adsorbed	0	0	3	49
Surface roughness	304	279	254	140

Relative adsorptions and roughnesses. Plating with two organic levelers, A and B, added to solution singly and together.



ticular satisfaction to him that he was able to take up his father's ideas and present them in the guise of a relation of complementarity between a purely physical and chemical account of biological phenomena and the use of the teleological concept of biological function equally indispensable for a complete description. Last summer, he had eagerly seized an opportunity to return to these considerations, in the light of the new discoveries in molecular biology. He had accepted an invitation to deliver an address at the opening of a new institute of genetics in Cologne, and he worked very hard at the preparation of this lecture. He sought the expert help of his younger Copenhagen colleagues, who, coming for the first time in closer contact with the venerated master, were struck by the candor and humility with which he submitted to their criticism. This paper was the one he was most anxious to finish. We are only left, alas, with a preliminary version of it, which, imperfect as it is, embodies his last thoughts on these fundamental questions.

Bohr had great expectations about the future role of complementarity. He upheld them with unshakable optimism, never discouraged by the scant response he got from our unphilosophical age. On one of those unforgettable strolls during which Bohr would so candidly disclose his innermost thoughts. we came to consider that what many people nowadays sought in religion was a guidance and consolation that science could not offer. Thereupon Bohr declared, with intense animation, that he saw the day when complementarity would be taught in the schools and become part of general education; and better than any religion, he added, a sense of complementarity would afford people the guidance they needed. Utopian dreams? It is up to us to respond to the challenge and make of today's Utopia the reality of tomorrow. We owe Bohr a tremendous widening of our scientific and philosophical horizon; he has proposed to us a view of the world of greater wealth and deeper harmony, and a conception of the function of science which makes its scope more universal and its appeal more human. But his most precious gift to us who have known him is the shining example of a life so earnestly devoted to truth, so full of wisdom and humanity. His name is written in the annals of history near those of Newton and Einstein; his place in our hearts is among those that are dearest to us.

These rough drawings were made by Niels Bohr on the blackboard of his working room at Carlsberg in the course of conversation on the eve of his death. The upper one represents a contour in a complex plane, surrounding a singularity (at the origin) of the nonuniform function  $\sqrt{Z}$ . It illustrates the early considerations on the ambiguity of language mentioned in the text. The lower drawing is a sketch of the "Einstein box", also described in the text. Fate has thus given us, by Bohr's own hand, a last symbolic record, as it were, of the beginnings and culmination of his thoughts on complementarity.

