

developments of concepts in

Nobel Laureate John Bardeen, professor of physics at the University of Illinois, presented the following address in London, England, on September 17, 1962, when he received the third Fritz London Award in recognition of his work in developing a successful theory of superconductivity. The presentation was made during the Eighth International Conference on Low Temperature Physics, which was held at Queen Mary College, University of London. The Proceedings of that conference, to be published by Butterworths, London, are expected to be available in April.

SUPERCONDUCTIVITY

By John Bardeen

IN this talk, I would like to trace the origins of some of the major theoretical and experimental advances which have led to the development of a successful microscopic theory of superconductivity and then to indicate briefly some of the current problems.¹ Fritz London made extremely important contributions to the theory and laid the groundwork for our present understanding. He showed how the Meissner effect and other superconducting properties could be understood as a consequence of quantum effects operating on a macroscopic scale. A striking prediction of his, which only recently has been verified experimentally, is the quantization of flux in a superconducting ring. My own work on superconductivity theory has been influenced greatly by that of Fritz London.

Progress in physics in this day and age is usually a result of a cooperative effort of large numbers of people working in different countries throughout the world, and this is certainly true of superconductivity. The impressive increase in understanding which has come about during the past few years is based on the work of many people. I particularly regret that my colleagues L. N. Cooper and J. R. Schrieffer, who collaborated on the basic work on the microscopic theory, are not able to share officially in the Fritz London Award. Both made essential contributions to what was truly a collaborative effort.

For more than twenty years after its discovery by Kammerlingh Onnes in 1911, superconductivity

was thought to be simply a case of infinite conductivity. Onnes himself carried out ingenious experiments with persistent current loops which illustrate very strikingly this aspect of the phenomenon. Other experiments at Leiden, by Keesom and co-workers, showed that superconductivity is associated with a phase transition of the electronic structure, which, in the absence of a magnetic field, is of second order with a jump in specific heat, but no latent heat at the transition temperature, T_c .

It was not until 1933 that Meissner and Ochsenfeld showed that a superconductor is a perfect diamagnet such that the magnetic field vanishes in all but a thin penetration region near the surface of a bulk specimen. Even when cooled into the superconducting state in the presence of a magnetic field, the flux is expelled from the interior. There is a unique current distribution in a simply connected body in the presence of a magnetic field, and these currents are stable, rather than metastable. The Meissner effect gives justification for application of thermodynamics to derive relations between critical fields and specific heats, as was done with great success by Keesom, Rutgers, Gorter, and others. For many years the equations derived from thermodynamics were the only reliable theoretical relations known for superconductors.

Since the explanation for superconductivity depends very essentially on quantum theory, there was no possibility for deriving a microscopic theory prior to 1926. Soon after 1926, quantum me-



The certificate accompanying the Fritz London Award is presented to John Bardeen (right) by the chairman of the Third Fritz London Award Committee, John R. Pellam of the California Institute of Technology.

chanics was applied successfully by Sommerfeld, Bloch, and others to account for most properties of normal metals in terms of a one-electron model. In this picture, it is assumed that each electron moves independently in a self-consistent field determined by the ions and other conduction electrons. Only during the past few years have we begun to understand why the one-electron model works as well as it does in spite of correlations resulting from large Coulomb forces acting between electrons. However, this model failed to account for superconductivity. Since the Meissner effect was unknown at the time, the early attempts were to account for the infinite conductivity, which would require the absence of scattering or, in analogy with a ferromagnet, stable or metastable distributions of currents. Bloch, in a famous theorem later extended by Bohm to many-body systems, showed that in the absence of a magnetic field the most stable system state of an electron system is that of zero current. Because of the frustrations which the many theorists who worked on the problem encountered, Bloch jokingly proposed a second theorem, that "any theory of superconductivity can be refuted."

In the absence of a fundamental theory, attempts were made to develop phenomenological theories to describe the behavior of superconductors. Of these, the most successful have been the two-fluid model of Gorter and Casimir to account for the thermal properties and the famous equations given by the

London brothers to account for the electromagnetic properties. Both theories were developed soon after the discovery of the Meissner effect. In the two-fluid model, one assumes that a fraction of the electrons are condensed into the ground state and take part in superfluid flow, while the rest behave normally and contribute to the specific heat. The London equations were designed to give the response to electric and magnetic fields, and they describe the diamagnetic aspects as well as those associated with infinite conductivity.

In a discussion on superconductivity held here in London at a meeting of the Royal Society in May 1935, Fritz London² suggested a way in which the phenomenological equations might follow from quantum theory. He took the point of view that the diamagnetic aspects are most basic and suggested that the entire superconductor behaves as a "single big diamagnetic atom". He supposed that "the electrons be coupled by some form of interaction in such a way that the lowest state may be separated by a finite interval from the excited ones". This may be the earliest suggestion of an energy gap. He then went on to give his familiar argument that if the ground state eigenfunction is "rigid" and thus not modified very much by an applied magnetic field, the current density will be proportional to the vector potential (in a suitable gauge), and thus give the London equation which describes the Meissner effect.

In later publications, he made the picture more concrete. His ideas as to the nature of superconductivity are well expressed in the last chapter of his book³ on the subject, written in 1950, just prior to the development of theories based on electron-phonon interactions. Some further quotations may be in order to show how close his ideas are to the present microscopic theory. With the diamagnetic approach, he states that "in thermal equilibrium there is *no permanent current* in an isolated superconductor *except in the presence of an applied magnetic field*, and there is no conservation of these currents; they differ for every variation of the strength or direction of the applied field". In a multiply connected superconductor, such as a ring, one does have conservation of flux, but the persistent current which gives the flux is metastable rather than stable. London then goes on to point out that "the long-range order of the (average) momentum is to be considered one of the fundamental properties of the superconducting state". This would be "due to the *wide extension in space* of the wave functions representing the same momentum distribution throughout the whole metal in

the presence as well as the absence of a magnetic field". It is "a quantum structure on a macroscopic scale" which requires "a kind of solidification or *condensation of the average momentum distribution*". These statements give an excellent characterization of the present microscopic theory. It is the common momentum of the paired electrons which gives the rigidity to the momentum distribution.

It seems to me that most of those who thought long and hard about superconductivity prior to the discovery of the Meissner effect in 1933 never got over an inner feeling that the really fundamental property of a superconductor is infinite conductivity or persistent currents, and this colored the way they thought about the subject in future years. While an adequate theory must explain both aspects, the diamagnetic approach has been the most fruitful in indicating the nature of the superconducting state.

Experimental methods for investigating the electrodynamic properties are difficult and have required a long period for development. One of the most significant quantities is the penetration depth, λ , which is a measure of the average depth of penetration of a magnetic field into the surface of a bulk specimen. Estimates of λ were first made in 1938, by von Laue, from measurements by Pontius of the critical fields of thin lead wires. However, the first direct measurements of penetration effects were those of Shoenberg, who, in 1939, measured the temperature variation of the magnetic susceptibility of mercury colloids, with particle size between 10^{-6} and 10^{-5} cm. These measurements indicated an increase in λ with temperature consistent with the law:

$$\left(\frac{\lambda(T)}{\lambda(0)}\right)^2 = 1 / \left[1 - \left(\frac{T}{T_c}\right)^4\right],$$

which follows from London theory if the concentration of superconducting electrons is presumed to vary with temperature as predicted by the Gorter-Casimir two-fluid model. This semiempirical law has been used ever since for analysis of penetration depths from experiment. Some of the methods used, such as that of Casimir, measure changes of λ with temperature, but λ is uncertain up to an additive constant. In this case the empirical law is used to estimate λ itself. Casimir's method, which involves measurements of the change in inductance of a coil closely wrapped around a superconducting cylinder, was first successfully applied by Laurmann and Shoenberg in 1947. Present theory indicates that there should be small departures from

this law at low temperatures, and such departures have been observed, although generally they are not as great as predicted by theory.

Another method for studying the electrodynamics which has been exceedingly fruitful was suggested by Heinz London in 1940. If the magnetic field varies with high frequency, there is, by Maxwell's equation, an associated electric field in the penetration region which can accelerate the normal electrons and give rise to a loss. London measured the surface resistance, and found that R_s is not discontinuous at T_c , but drops rapidly from the value in the normal state, and approaches zero as $T \rightarrow 0$. After the war, this method was taken up by Pippard and others, who extended the measurements to obtain surface reactance as well as surface resistance at microwave frequencies. The reactance gives more direct information about penetration depths.

From an extensive series of measurements, including studies of effects of a static magnetic field and of alloying on penetration depths, Pippard suggested his so-called nonlocal modifications of the London equations in which a coherence distance, $\xi_0 \sim 10^{-4}$ cm, is introduced. According to Pippard, the current density is not proportional to the vector potential, A , but is determined by an integral of A over a region with dimensions of $\sim \xi_0$ surrounding the point in question. Present microscopic theory leads to an expression for the current similar to that suggested by Pippard, and subsequent experiments have given strong confirmation of the nonlocal version. Pippard also studied the frequency dependence of the surface impedance, and found results which could not be interpreted in any simple way in terms of a two-fluid model. The present theory has resolved most of these difficulties and gives good ($\sim 15\%$) agreement with recent experiments, such as those of Biondi and Garfunkel on aluminum, of Pippard and of Kaplan et al. on tin, and of Khaikin on cadmium. Some puzzling features of the dependence of surface impedance on magnetic field, as observed by Pippard, by Spiewak, and by Richards, have not as yet been accounted for by the theory.

We turn now to trace some of the ideas which led to our present picture of an excitation spectrum of a superconductor with an energy gap for excitation of electrons out of the ground state. A powerful method of approach in low-temperature physics is to consider first the nature of the state at the absolute zero and then the elementary thermal excitations. Examples are spin waves in ferromagnetism, phonons, Landau's rotons in liquid helium, and quasi-particle excitations of electrons from the

Fermi sea in superconducting and normal metals. The method of elementary excitations has been emphasized and exploited by Landau in his theory of superfluid He II and in his theory of a Fermi liquid. It was for these and other noteworthy achievements that Landau received the Fritz London award of 1960.

There have been several suggestions through the years that a superconductor has a quasi-particle spectrum with a gap. As we have seen, this suggestion was made as early as 1935 by London. This picture was developed more completely in 1938 by Welker in an attempt to account for the Meissner effect. Daunt and Mendelssohn (1946) observed that there is no Thompson heat associated with supercurrent flow and that therefore no entropy is transported with the supercurrent. They suggested a gap to excited states which contribute to the electronic specific heat. Ginzburg and others have developed two-fluid models with a gap to account for the thermal properties. Earlier, Koppe had proposed a two-fluid model, based loosely on the Heisenberg-Koppe theory of superconductivity, which could be interpreted in terms of a temperature-dependent energy gap.

Experimentally, information about the spectrum of elementary excitation can be obtained from specific heats and from various transport phenomena, such as thermal conductivity, surface impedance, attenuation of ultrasonic waves, and, more recently, from transmission of electromagnetic radiation through thin films, nuclear spin relaxation times, and tunneling experiments. If there is a gap, the number of quasi-particle excitations at low temperatures should vary as $\exp[-E_g/2kT]$, where E_g is the gap. The first convincing experimental evidence for such an exponential temperature dependence came from Goodman's measurements of the thermal conductivity of tin. He found good agreement with the Koppe theory, which, as Goodman pointed out, could be interpreted as an energy-gap model.

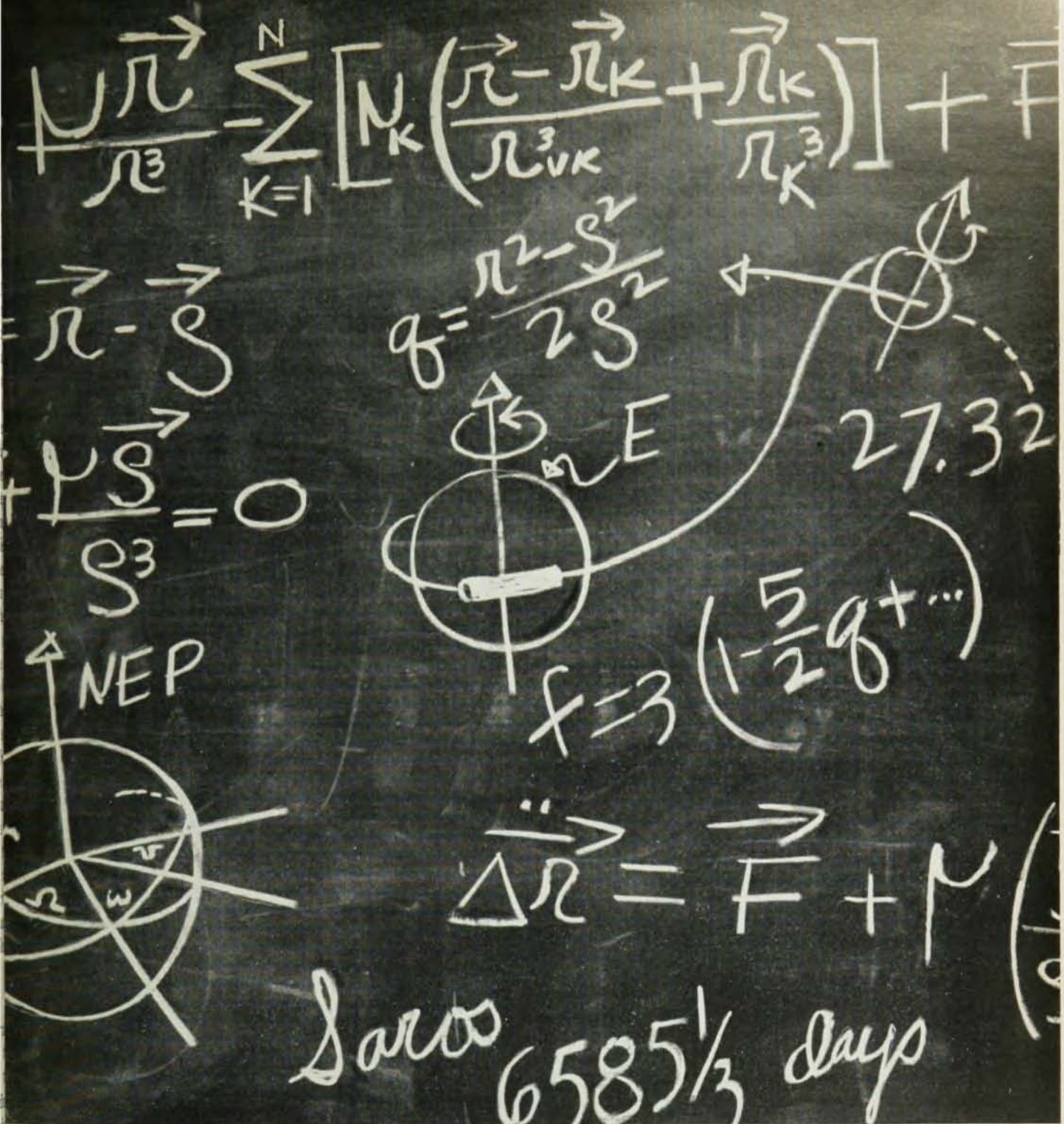
Early measurements of specific heats were in approximate agreement with the Gorter-Casimir T^3 law, which corresponds to a reduced density of states but not to a true gap at low temperatures. The first measurements which showed a marked deviation from this law and an exponential temperature dependence were those of Brown, Zemansky, and Boorse on niobium. A little later, careful specific-heat measurements by Corak and coworkers on vanadium and on tin gave good evidence for the exponential behavior.

If there is a gap, there should, at very low tem-

peratures, be no absorption of electromagnetic radiation unless the energy quantum $h\nu$ exceeds the gap. Unfortunately, for most superconductors the critical frequency is in the most difficult part of the spectrum to work in, the very far infrared just beyond the microwave range. Prior to 1956, surface resistance experiments had shown that the energy absorption goes to zero as $T \rightarrow 0$ in the microwave range and that there is no difference in reflectivity, and thus in absorption, between normal and superconducting states in the infrared. There were no measurements at intermediate frequencies, so that it was uncertain at which frequency absorption would start. By developing techniques to work in the very far infrared, Tinkham and Glover were able to fill in the missing part of the spectrum and determine the critical frequency. In their first experiments, they measured the transmission through very thin films of lead and tin. These were analyzed to give the real and imaginary part of the complex conductivity, $\sigma = \sigma_1 - i\sigma_2$. They found that near $T = 0^\circ\text{K}$, σ_1 increases abruptly from zero when ν becomes greater than a critical frequency and then rapidly approaches the normal conductivity at higher frequencies. The experiments thus give a direct measure of the gap, which they found to be about $3.5 kT_c$ for tin and $4.0 kT_c$ for lead. Their experiments in the microwave range also gave excellent confirmation of the nonlocal theory of Pippard and an estimate of the coherence distance ξ_0 in fair agreement with values given by Pippard and Faber for the same metals. The first experiments of Tinkham and Glover were done just prior to the development of the microscopic theory. These and subsequent experiments on absorption of electromagnetic radiation in the far infrared by Tinkham, Ginsberg, and Richards have provided an excellent experimental test of the microscopic theory.

WE have discussed some of the most important experiments which have played a role in the development of phenomenological theories of the electrodynamic properties and of various two-fluid models for the thermal properties of superconductors. We now turn to a discussion of some of the theoretical concepts and experiments which have been important in the development of the microscopic theory. This will be a more personal account in that I will confine the discussion to ideas which have had an important influence on my own thinking about the subject.

My first attempt to construct a theory of superconductivity was made in the late thirties and was strongly influenced by London's picture, outlined



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earlier. I thought that it might be possible to extend the Bloch one-electron model to account for superconductivity. A periodic potential introduces Brillouin-zone boundaries in k space, with an energy gap at the boundary proportional to the Fourier coefficient of the potential. If one could produce zone boundaries at nearly all parts of the Fermi surface, one would get a lowering of energy of the electrons in states just inside the surface. No matter how complex the Fermi surface, it should be possible to accomplish this by introducing many small periodic distortions of the lattice corresponding to a very large complex unit cell. The attempt to construct a theory along these lines was not successful; various objections were raised. Further, more accurate estimates showed that this type of instability is unlikely to occur in real metals at low temperatures. The work was interrupted by the war and all that was published was an abstract of the talk. Much later, Fröhlich developed a far more complete theory for a one-dimensional model along similar lines.

After the war, my research interests turned to semiconductors, and it was not until May 1950, when I heard about the isotope effect from Serin, that I resumed work on superconductivity theory. Separated isotopes became available after the war, so that it was possible to determine whether or not there was a dependence of critical temperature on isotopic mass. Experiments were undertaken independently by Reynolds, Serin et al. at Rutgers and by Maxwell at the National Bureau of Standards, first on mercury. These showed, surprisingly at the time, that T_c varies inversely with the square root of the isotopic mass. The mass would not be an important parameter unless the motion of the ions is involved, which suggested that superconductivity must arise from some sort of interaction between the electrons and zero-point vibrations of the lattice. I attempted to develop a theory in which I suggested that the effect of the interaction would be such as to lower the energy of electrons near the Fermi surface, but as a result of dynamic interactions with the zero-point motion rather than by periodic lattice distortions.

About a week after I sent in a letter to the editor outlining these ideas, Fröhlich visited the Bell Telephone Laboratories where I was working at the time. He told me about his own work on a theory of superconductivity based on electron-phonon interactions, which he had done at Purdue in the spring of 1950. Fröhlich's work was done without knowledge of the isotope effect. He was greatly encouraged when he learned, just about the time he

was ready to send his manuscript to *The Physical Review*, about this strong experimental confirmation of his approach. Although there were mathematical difficulties in both his approach and mine, primarily because of a use of perturbation theory in a region where it is not justified, we were both convinced that at last we were on the road to an explanation of superconductivity.

It did not take long to discover that the difficulties with these theories were basic and not easy to overcome. This was shown perhaps most clearly by a calculation of Schafroth, who contributed much to superconductivity theory. His untimely death cut short a promising career. Schafroth showed that a theory based on treating the electron-phonon interaction by perturbation theory could not account for the Meissner effect, even though the expansion is carried to arbitrarily high order.

These theories of Fröhlich and myself were based essentially on the self-energy of the electrons in the phonon field rather than on a true interaction between electrons. It became evident that all or nearly all of the self-energy is included in the normal state and is not much changed by the transition.

In Fig. 1 we have reproduced a slide made in 1955 to illustrate the status of the theory up to that time. The thermal properties gave evidence for an energy gap for excitation of quasi-particle from the superconducting ground state. Further, I showed that if one assumed a reasonable energy-gap model, one could account for the Meissner effect, but with a nonlocal theory similar to that proposed by Pipard. The "derivation" of the Meissner effect which I gave at that time has been criticized by Buckingham and others on the grounds that the calculation is not gauge invariant, but I believe that the argument as given is essentially correct and is in accord with the present microscopic theory. The energy-gap model was the unifying theme of my review article which appeared in 1956 in *Handbuch der Physik*, Vol. XV. At that time there was no way to derive an energy-gap model from microscopic theory. While the Heisenberg-Koppe theory based on Coulomb interactions could be interpreted in terms of an energy gap, it did not yield the isotope effect and was also subject to other difficulties. Thus, at that time, it appeared that the main problem of the microscopic theory was to show how electron-phonon interactions might yield an energy gap.

That electron-phonon interactions lead to an effective attractive interaction between electrons by exchange of virtual phonons was shown by Fröhlich by use of field-theoretic techniques. His analysis was extended by Pines and myself to include Cou-

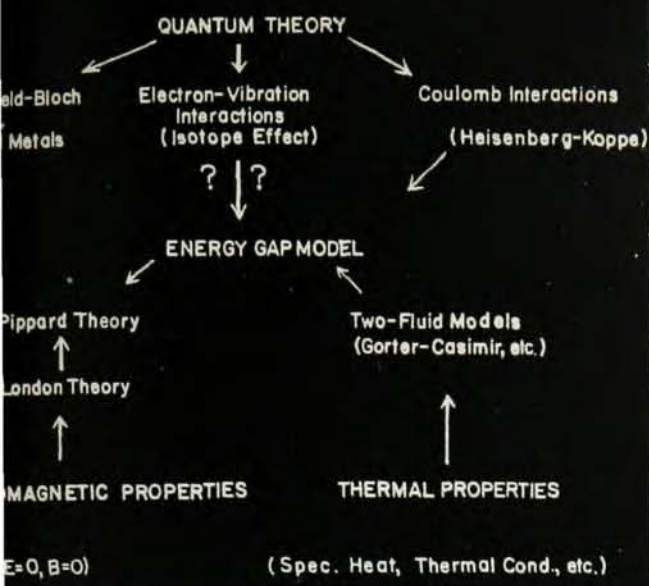


Fig. 1. Reproduction of a slide made in 1955 to illustrate the status of the theory at that time. Experiments on thermal properties gave evidence for an energy gap for excitation of electrons from the superconducting ground state. It was shown that a reasonable energy-gap model would most likely lead to Pippard's nonlocal modification of the phenomenological London equations to describe the electromagnetic properties. Thus, it seemed, the major problem was to see how an energy gap might follow from a microscopic theory based on interactions between electrons and phonons, as indicated by the isotope effect.

lomb interactions. In second order, there is an effective interaction between the quasi-particle excitations of the normal state which is the sum of the attractive phonon-induced interaction and a screened Coulomb interaction. In the *Handbuch* article, I suggested that one should take the complete interaction, not just the diagonal self-energy terms, and use it as the basis for a theory of superconductivity.

The next major step was made by Cooper, who, following up this approach, showed that if there is an effective attractive interaction, a pair of quasi-particles above the Fermi sea will form a bound state no matter how weak the interaction. If the binding energy is of the order of kT_c , the size of the pair wave function is of the order of 10^{-5} to 10^{-4} cm. This calculation showed definitely that, in the presence of attractive interactions, the Fermi sea which describes the ground state of the normal metal is unstable against the formation of such bound pairs. However, one could not use this calculation immediately to construct a theory of superconductivity. If all of the electrons within $\sim kT_c$ of the Fermi surface form such bound pairs, the spacing between the pairs would be only $\sim 10^{-6}$ cm, a distance much smaller than the size of a pair. Because of the considerable overlap between the pairs, and because of the exclusion principle and required antisymmetry of the wave functions, they cannot be regarded as moving independently. Thus, the picture proposed earlier by Schafroth (1955), and developed more completely in cooperation with Butler and Blatt, of electron pairs as "localized entities (pseudo-molecules) whose center-of-gravity motion is essentially undisturbed", and which at low temperatures undergo an Einstein-Bose condensation, is not valid. New methods were required to construct

a theory of superconductivity, and this was first accomplished by the joint efforts of Cooper, Schrieffer, and myself. While the theory can be and has been developed by use of a variety of mathematical techniques, I believe that the variational method used in our original publications gives as good a picture as any of the ground-state wave functions and of the quasi-particle excitation spectrum with a gap.

One may describe the low-lying configurations for the normal phase of a metal by specifying the occupancy in k -space of the quasi-particles above the Fermi sea and of unoccupied states or holes below the sea. In accordance with the Landau Fermi-liquid model, the energy of one quasi-particle may depend on the distribution of the other quasi-particles. These quasi-particle configurations are not exact solutions of the Hamiltonian when Coulomb and phonon interactions are included, but are reasonably well defined if the excitation energies are not too high. The configurations are presumed to include correlation energies and quasi-particle self-energies characteristic of the normal phase. Superconductivity arises from residual attractive interactions between the normal quasi-particles.

Cooper, Schrieffer, and I took for the variational wave-function ground state of a superconductor a linear combination of normal configurations in which the quasi-particle states are occupied in pairs ($k_1\uparrow, k_2\downarrow$) of opposite spin and the same total momentum, $k_1 + k_2 = q$, common to all pairs. In any configuration, the two states of a pair are either both occupied or both empty. Values of q different from zero describe current flow in the ground state; that for $q = 0$ for zero current has the lowest energy. We also worked out a quasi-particle excitation spectrum for a superconductor in one-to-one correspondence with that for a normal metal, with a temperature-dependent energy gap for excitation of particles from the superconducting ground state.

A superconductor differs from a semiconductor in that the gap in the former is relative to the Fermi surface and is not fixed in k -space. The entire system with interactions can be displaced in momentum space to give a net current flow.⁴ If v_s is the velocity of flow, the mass flow at $T = 0^\circ\text{K}$ is ρv_s , where $\rho = nm$ is the density of the electrons. At a finite temperature, quasi-particle excitations will reduce the current, but when a local equilibrium is established corresponding to a given v_s , a net flow $\rho_s v_s$ will remain. This defines the density of the superfluid component of the two-fluid model, ρ_s . With increasing temperature, ρ_s decreases from ρ at $T = 0^\circ\text{K}$ to zero as $T \rightarrow T_c$. When the Fermi sea of a normal metal is displaced in momentum space, quasi-particle excitations soon reduce the current to zero, so that $\rho_s = 0$. A superfluid is characterized by a value of ρ_s different from zero. These considerations are analogous to those Landau used to account for the superfluidity of liquid helium.

The metastability of persistent currents does not occur because of lack of scattering. Quasi-particles are readily scattered, but such scattering does not change the common momentum of the pairs and thus v_s . It only results in fluctuations about the current corresponding to local quasi-particle equilibrium, $\rho_s v_s$.

The theory has been applied to a wide variety of properties such as specific heats, electromagnetic properties, thermal conductivity, ultrasonic attenuation, nuclear spin relaxation times, the Knight shift and electron spin paramagnetism, electron tunneling, critical fields and currents, boundary effects, and other problems. In nearly all cases excellent agreement between theory and experiment is found when the parameters of the theory are evaluated empirically. Difficulties associated with thermal conductivity for phonon scattering and with the Knight shift appear to be on the way to resolution through a combination of experimental and theoretical work.

An unexpected feature of the theory is the marked effect of coherence on the matrix elements for scattering of quasi-particles in a superconductor. It accounts for phenomena which would be inexplicable on the basis of any simple two-fluid model. In the early spring of 1957, when Cooper, Schrieffer, and I were first working out the details of the theory, Hebel and Slichter, also working at Illinois, made the first measurements of nuclear-spin relaxation times in a superconductor by use of ingenious experimental techniques. They found, surprisingly, a marked *decrease* in the relaxation time as the temperature dropped below T_c in the

superconducting state, followed by an increase at still lower temperatures. Relaxation of the nuclear spins occurs from interaction with the conduction electrons in which there is a spin flip of the electron as well as the nucleus. The experiments indicated a *larger* interaction in the superconducting than in the normal state, even though specific heats and other experiments showed that there must be a marked decrease in the number of quasi-particle excitations as the temperature drops below T_c . For example, the attenuation of ultrasonic waves drops abruptly at T_c . These apparently contradictory experiments are accounted for by coherence effects. In calculating matrix elements for quasi-particle transitions in a superconductor, we found that it is necessary to add coherently the contributions from electrons of opposite spin and momentum in the various normal configurations which make up the quasi-particle states of a superconductor. For the case of a spin flip, the two contributions to the matrix element add constructively, and the larger transition probability in the superconducting state is a result of the increased density of states in energy. For an ordinary interaction such as occurs in ultrasonic attenuation, the contributions add destructively, giving a drop with an infinite slope at T_c , as observed. The experimental check of these very marked effects of coherence provides one of the best confirmations of pairing in the wave functions.

In working out the properties of our simplified model and comparing with experimental results on real metals, we were continually amazed at the excellent agreement obtained. If there was serious discrepancy, it was usually found on rechecking that an error was made in the calculations. Everything fitted together neatly like the pieces of a jigsaw puzzle. Accordingly, we were unprepared for the skepticism with which the theory was greeted in some quarters. Those most skeptical had generally worked long and hard on superconductivity theory themselves, and had their own ideas of what the theory should be like. Most of the criticism centered on our derivation of the Meissner effect, because it was not carried out in a manifestly gauge-invariant manner. While our derivation is not mathematically rigorous, we gave what we believe are good physical arguments for our use of a transverse gauge, and our procedure has been justified in subsequent work. As we have seen, our model is exactly of the sort which should account for superconductivity according to London's ideas.

At the opposite extreme were some who felt that the explanation of superconductivity would mark

the end of what had long been a puzzling and challenging scientific problem. On the contrary, the theory has stimulated much new experimental and theoretical work; it has helped put new life into the field. While some questions have been answered, many others have been raised as we probe more deeply, and plenty of problems remain, as is evident from the papers submitted to this meeting.

Since the original publications, the mathematical formulation of the theory has been developed considerably. Several different mathematical formulations have been given which have improved the rigor and have extended the theory so as to apply to a wider variety of problems. Particular mention should be made of the work of Bogoliubov and co-workers, who, along with Valatin, introduced the now famous transformation to quasi-particle variables, gave a much improved treatment of Coulomb interactions, provided a treatment of collective excitations, and made other noteworthy contributions. Independently of this work, Anderson gave a derivation based on an equation-of-motion approach which introduced collective excitations and allowed a manifestly gauge-invariant treatment of the Meissner effect. The approaches of Bogoliubov and of Anderson were extended by Rickayzen to give probably the most complete derivation of the Meissner effect to date. Green's-function methods, borrowed from quantum field theory, have been used widely and with great success, following the initial work of Gor'kov, Martin and Schwinger, Kadanoff, and others. Gor'kov, in particular, has used these methods to solve several difficult problems in superconductivity theory. Fröhlich was one of the pioneers in the use of field-theoretic methods in solid-state problems.

THE theory of superconductivity also has stimulated a great deal of new experimental work. Some of these experiments have been made to test various predictions of the theory, particularly those associated with the energy gap. Of the really new experiments, the most remarkable are those on electron tunneling, first done by Giaever, and the observation of flux quantization by Deaver and Fairbank and by Doll and Näbauer. Refined measurements have been made in various laboratories of such things as penetration depths, surface impedance, thermal conductivity, and ultrasonic attenuation. These experiments have provided critical tests of the theory and have indicated directions in which improvements are desired. Experiments on nuclear-spin relaxation, particularly by Redfield and associates, and on far-infrared transmission and reflection

by Tinkham and associates have continued to be very fruitful.

An area of study which is receiving increasing attention from both experimentalists and theorists is that of boundary effects, such as the normal-superconducting boundary in the intermediate state and the boundary between a superconductor and non-superconductor. Closely related are the problems of critical currents and fields in thin films or other specimens of small dimensions. In 1950, long before the development of the microscopic theory, Ginzburg and Landau proposed a phenomenological extension of the London equations to treat such problems. Gor'kov showed that the microscopic theory leads to the Ginzburg-Landau equations near T_c , where the penetration depth is so large that local London theory may be applied. He also derived the much more complicated equations which are required when the Pippard nonlocal version of the theory must be used. The only difference is that an effective charge $2e$, representing that of a pair, appears in place of e in the original version of Ginzburg-Landau. This change improves agreement between theory and experiment. An effective charge $2e$ also appears in the magnitude of the flux quantum, giving a value half of that predicted by London.

A derivation from first principles of the effective interaction which gives rise to superconductivity, and thus the critical temperature, is a very difficult task and has not been accomplished for any superconducting element, compound, or alloy. Thus far, the theory has little to say about the very basic question of the occurrence of superconductivity among the elements of the periodic table, and is unable to predict which compounds or alloys might become superconducting. Some rather rough calculations by Pines and Morel on the basis of a simplified model are in qualitative agreement with the empirical rules of Matthias. Matthias, together with Hulm and others, has studied the occurrence of superconductivity in a large number of substances, and has found many new superconducting compounds and alloys. Among these are Nb_3Sn , V_3Ga , and other compounds which are so promising for use in high-field magnets. Work of Matthias and coworkers on the coexistence of superconductivity and ferromagnetism in the same substance, on the effects of ferromagnetic impurities with localized moments, and on related problems raises many questions which so far have not had extensive theoretical treatment. By being careful to remove all traces of ferromagnetic impurity, Geballe, Matthias, and coworkers recently have found three new su-

perconducting elements, Mo, Ir, and Lu. Perhaps their most interesting discovery is the absence of an isotope effect in Ru and Os. In a paper presented at this meeting, they report observations of an isotope effect in Mo about two-thirds as large as that found in nontransition elements, showing that electron-phonon interactions are important for this element. These experiments suggest that an effective attractive interaction may arise from some mechanism other than electron-phonon interactions in the transition elements.

There is also a problem about the isotope effect in nontransition-metal superconductors. Here the question is why the exponent α in the relation $T_c \sim M^{-\alpha}$ is always observed to be so close to 0.5. Refined treatments of the effect of Coulomb interactions along the lines initially suggested by Bogoliubov indicate that α should depart significantly from 0.5, contrary to what is found.

We shall probably see more experimental and theoretical work done on anisotropic effects in superconductors associated with the structure of the Fermi surface. Much has been learned in recent years about the Fermi surface from studies of various properties of normal metals which should form a basis for work on superconducting properties.

Before concluding, I would like to say a few words about the impact of superconductivity theory on other branches of physics. Following the initial work of Bohr, Mottelson, and Pines, the theory has been applied with great success to nuclear structure and has created almost as much of a revolution in understanding in this field as it has in superconductivity in metals. We now speak of superfluid nuclei and of the energy gap in the nucleus. With the effective interaction which gives the gap described by an empirical parameter which varies slowly through the periodic system, one can account for a wide variety of experimental results. I was tremendously impressed by the progress made when I learned about some of this work during the past summer from lectures of Baranger at the

French Summer School at Cargèse, Corsica, and from various workers in the field at a small conference on superconductivity and nuclear structure at Chaumont, Switzerland. Nuclear theorists have been able to account quantitatively for pairing energies, nuclear spectra, moments of inertia, nuclear shapes, and otherwise unexplained features of α - and of β -decay.

It has been suggested that there should be a transition to a superfluid phase in liquid He³, but experiments made thus far to temperatures below .01°K give no indication of such an effect.

While solid-state theorists have borrowed techniques from quantum field theory, field theorists have been attempting to apply some of the methods of superconductivity theory to the elementary particles of high-energy physics. The vacuum corresponds to the ground state and the particles to the elementary excitations. Nambu has proposed a theory in which the masses of nucleons arise in much the same way as the energy gap. A more ambitious attempt along similar lines has been suggested by Fisher. Dürr and Heisenberg have used some of the concepts, including that of a degenerate vacuum, in their theory of elementary particles. We will probably hear much more about these developments in the years to come. While for many years solid-state, nuclear, and field theorists followed divergent paths, there is now an underlying unity, which we like to think is characteristic of physics.

We have seen that the development of our understanding of superconductivity has resulted from a close interplay of theory and experiment. Physical insight into the nature of the superconducting state gained from a study of the experimental findings has been essential to make progress in the theory. Increased theoretical understanding has suggested new experiments, new paths to explore, and has helped to understand better such seemingly unrelated fields as nuclear structure and elementary particles.

References

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4. To simplify the argument, we omit effects of the magnetic field on current paths, which is not valid except for flow in very thin films.