

we will continue to put the atomic mass number in the right superscript position, e.g., ${}^7\text{N}^{14}$. While adopting the SUN table of prefixes to represent the various powers of ten, we accept either G or B as an abbreviation for 10^9 .

The issue of whether to say kilocycle when we mean kilocycle per second (with abbreviations kc and kc/sec) is unresolved. Where temperature intervals, rather than

temperatures on a scale, are meant, the use of $^{\circ}\text{C}$, $^{\circ}\text{F}$, $^{\circ}\text{K}$ or deg C, deg F, deg K is acceptable.

The author of this note is the US member of the SUN Commission and is the chairman of the NAS-NRC Committee on Symbols, Units and Nomenclature. He will welcome recommendations from physicists for further progress toward standardization in these areas.

Hugh C. Wolfe

AIP Director of Publications

SYMBOLS UNITS and NOMENCLATURE IN PHYSICS

INTRODUCTION

The recommendations in this document, composed by the Commission for Symbols, Units and Nomenclature (SUN Commission) of the International Union of Pure and Applied Physics (IUPAP), have been approved by the successive General Assemblies of the IUPAP, held in 1948, 1951, 1954, 1957 and 1960.

This document replaces the documents: SUN 49-1, SUN 53-1, UIP 6 (1955) and SUN 57-9, in which the previous recommendations of the SUN Commission were presented.

The recommendations contained in this document are in general in agreement with recommendations of the following international organizations:

1. International Organization for Standardization, Technical Committee ISO/TC 12;
2. General Conference on Weights and Measures (1948, 1954, 1960);
3. International Union of Pure and Applied Chemistry;
4. International Electrotechnical Commission, Technical Committees IEC/TC 24, 25;
5. International Commission on Illumination.

Prof. P. Fleury, *Secretary General*,
International Union of Pure and Applied Physics,
3 Boulevard Pasteur, Paris 15, France

Prof. J. de Boer, *Secretary*,
Commission for Symbols, Units and Nomenclature,
Roetersstraat 1a, Amsterdam (C), Netherlands

1. Physical Quantities—General Recommendations

1.1 The symbol for a *physical quantity* (French: 'grandeur physique', German: 'physikalische Grösse', American, sometimes: 'physical magnitude') is equivalent to the product of the *numerical value* (or the measure), a pure number, and a *unit*, i.e.,

$$\text{physical quantity} = \text{numerical value} \times \text{unit}.$$

For dimensionless physical quantities the unit often has no name or symbol and is not explicitly indicated.

Examples: $E = 200 \text{ erg}$ $n_{\text{qu.}} = 1.55$
 $F = 27 \text{ N}$ $\nu = 3 \times 10^8 \text{ s}^{-1}$

1.2 Symbols for physical quantities—General rules

1. Symbols for physical quantities should be single letters of the Latin or Greek alphabet with or without modifying signs: subscripts, superscripts, dashes, etc.

Remark:

a. An exception to this rule consists of the two letter symbols, which are sometimes used to represent dimensionless combinations of physical quantities. If such a symbol, composed of two letters, appears as a factor in a product, it is recommended to separate this symbol from the other symbols by a dot or by brackets or by a space.

b. Abbreviations, i.e. shortened forms of names or expressions, such as p.f. for partition function should not be used in physical equations. These abbreviations in the text should be written in ordinary Roman type.

2. Symbols for physical quantities should be printed in *italic* (or *sloping*) type.

Remark:

It is recommended to consider as a guiding principle for the printing of indices the criterion: only indices which are symbols for physical quantities should be printed in italic (sloping) type. Examples:

Upright indices	Sloping indices
C_g (g = gas)	p in C_p
g_n (n = normal)	n in $\sum_n a_n \varphi_n$
μ_r (r = relative)	x in $\sum_x a_x b_x$
E_k (k = kinetic)	i, k in g_{ik}
χ_e (e = electric)	x in p_x

3. Symbols for vectors and tensors: To avoid the usage of subscripts it is often recommended to indicate vectors and tensors of the second rank by letters of a special type. The following choice is recommended:

- Vectors should be printed in bold type, by preference bold italic (sloping) type, e.g. ***A***, ***a***.
- Tensors of the second rank should be printed in sans serif type, e.g. **S**, **T**.

Remark:

When this is not possible, vectors may be indicated by

an arrow and tensors by a double arrow on top of the symbol.

1.3 Simple mathematical operations

1. Addition and subtraction of two physical quantities are indicated by:

$$a + b \quad \text{and} \quad a - b$$

2. Multiplication of two physical quantities may be indicated in one of the following ways:

$$ab \quad a b \quad a \cdot b \quad a \cdot b \quad a \times b$$

Remark: The various products of vectors and tensors may be written in the following ways:

scalar product of vectors **A** and **B**: ***A* · *B*** ***A* · *B***
 vector product of vectors **A** and **B**: ***A* ∧ *B*** ***A* × *B***
 dyadic product of vectors **A** and **B**: ***AB***

scalar product of tensors **S** and **T** ($\sum_{i,k} S_{ik} T_{ki}$) ***S* · *T***
 tensor product of tensors **S** and **T** ($\sum_{i,k} S_{ik} T_{kl}$) ***S* · *T* *S* · *T***
 product of tensor **S** and vector **A** ($\sum_k S_{ik} A_k$) ***S* · *A* *S* · *A***

3. Division of one quantity by another quantity may be indicated in one of the following ways:

$$\frac{a}{b} \quad a/b \quad a \, b^{-1}$$

or in any other way of writing the product of a and b^{-1} .

These procedures can be extended to cases where one of the quantities or both are themselves products, quotients, sums or differences of other quantities.

If necessary brackets have to be used in accordance with the rules of mathematics.

If the solidus is used to separate the numerator from the denominator and if there is any doubt where the numerator starts or where the denominator ends, brackets should be used.

Examples:

Expressions with a horizontal bar	Same expressions with a solidus
$\frac{a}{bcd}$	a/bcd
$\frac{2}{9} \sin kx, \frac{1}{2} RT$	$(2/9) \sin kx, (1/2) RT$ or $RT/2$
$\frac{a}{b} - c$	$a/b - c$
$\frac{a}{b - c}$	$a/(b - c)$
$\frac{a - b}{c - d}$	$(a - b)/(c - d)$
$\frac{a}{c} - \frac{b}{d}$	$a/c - b/d$

Remark: It is recommended that in expressions like:

$$\sin\{2\pi(x - x_0)/\lambda\} \quad \exp\{(r - r_0)/\sigma\} \\ \exp\{-V(r)/kT\} \quad \sqrt{\epsilon/c^2}$$

the argument should always be placed between brackets, except when the argument is a simple product of two quantities: e.g. $\sin kx$. When the horizontal bar above the square root is used no brackets are needed.

2. Units—General Recommendations

2.1 Symbols for units—General rules

1. Symbols for units of physical quantities should be printed in *roman (upright) type*.

2. Symbols for units should not contain a final full stop and should remain unaltered in the plural, e.g.: 7 cm and *not* 7 cms.

3. Symbols for units should be printed in *lower case roman (upright) type*. However, the symbol for a unit, derived from a proper name, should start with a capital roman letter, e.g.: m (metre); A (ampere); Wb (weber); Hz (hertz).

2.2 Prefixes—General rules

1. The following prefixes should be used to indicate decimal fractions or multiples of a unit.

deci (=10 ⁻¹)	d		
centi (=10 ⁻²)	c		
milli (=10 ⁻³)	m	kilo (=10 ³)	k
micro (=10 ⁻⁶)	μ	mega (=10 ⁶)	M
nano (=10 ⁻⁹)	n	giga (=10 ⁹)	G
pico (=10 ⁻¹²)	p	tera (=10 ¹²)	T
femto (=10 ⁻¹⁵)	f		
atto (=10 ⁻¹⁸)	a		

2. The use of *double prefixes* should be avoided when single prefixes are available.

<i>Not:</i> m μ s,	<i>but:</i> ns (nanosecond)
<i>Not:</i> kMW,	<i>but:</i> GW (gigawatt)
<i>Not:</i> $\mu\mu$ F,	<i>but:</i> pF (picofarad)

3. When a prefix is placed before the symbol of a unit, the combination of prefix and symbol should be considered as *one new symbol*, which can be squared or cubed without using brackets.

Examples:

$$\text{cm}^2, \quad \text{mA}^2, \quad \mu\text{s}^2$$

Remark: A prefix should never be used before a unit which is squared. Thus:

$$\text{cm}^2 \text{ means always } (0.01 \text{ m})^2 \text{ and never } 0.01 \text{ m}^2.$$

2.3 Mathematical operations

1. Multiplication of two units may be indicated in one of the following ways:

$$\text{Nm} \quad \text{N m} \quad \text{N}\cdot\text{m} \quad \text{N.m}$$

2. Division of one unit by another unit may be indicated in one of the following ways:

$$\frac{\text{m}}{\text{s}} \quad \text{m/s} \quad \text{m s}^{-1}$$

or by any other way of writing the product of m and s⁻¹. Not more than one solidus should be used. *Examples:*

$$\text{Not: cm/s/s,} \quad \text{but: cm/s}^2 = \text{cm s}^{-2}$$

$$\text{Not: 1 poise} = 1 \text{ g/s/cm,} \quad \text{but: 1 poise} = 1 \text{ g/s cm} \\ = 1 \text{ g s}^{-1} \text{ cm}^{-1}$$

$$\text{Not: J/}^\circ\text{K/mol,} \quad \text{but: J/}^\circ\text{K mol} \\ = \text{J } ^\circ\text{K}^{-1} \text{ mol}^{-1}$$

3. Numbers and Figures

1. Numbers or figures should be printed in *upright type*.

2. The decimal sign between figures or numbers should be a comma (,) or (but *only in English texts*) a point.

3. The multiplication sign between figures or numbers should be a cross (×) or (but *only in non-English texts*) a point.

4. Division of one figure or number by another figure or number may be indicated in the following ways:

$$\frac{136}{273,15} \quad 136/273,15$$

or by writing it as the product of numerator and the inverse power of the denominator. In such cases the number under the inverse power should always be placed between brackets.

Remark: When the solidus is used and when there is any doubt where the numerator starts or the denominator ends, brackets should be used, as in the case of quantities (see 2.4).

5. To facilitate the reading of large numbers, the figures may be grouped in *groups of three*, but *no comma or point* should be used except for the decimal sign.

Example: 2 573, 421 736.

4. Symbols for Chemical Elements, Nuclides, and Particles

1. Symbols for chemical elements should be written in *roman (upright) type*. The symbol is not followed by a full stop.

$$\text{Examples: Ca C H He}$$

2. The attached numerals specifying a *nuclide* are:

mass number $^{14}\text{N}_2$ atoms/molecule

Remark: The atomic number may be placed as a left subscript, if desired. The right superscript position should be used, if required, for indicating a state of ionization (e.g. Ca^{2+} , PO_4^{3-}) or an excited state (e.g. $^{110}\text{Ag}^m$, He^*).

3. Symbols for particles and quanta.

neutron	n	pion	π
proton	p	muon	μ
deuteron	d	electron	e
triton	t	neutrino	ν
α -particle	α	photon	γ

It is recommended that the following notation should be used:

Hyperons: Upright capital Greek letters to indicate specific particles, e.g. Λ , Σ .

Nucleons: Upright lower case n and p to indicate neutron and proton respectively.

Mesons: Upright lower case Greek letters to indicate specific particles, e.g. π , μ , τ .

Leptons: L-particles; e.g. e, ν .

It is recommended that the charge of particles may be indicated by adding the superscript +, - or 0. *Examples:*

$$\pi^+, \pi^-, \pi^0, p^+, p^-, e^+, e^-.$$

If in connection with the symbols p and e no charge is indicated, these symbols should refer to the positive proton and the negative electron respectively.

The symbol $\sim$ above the symbol of a particle has been used to indicate the anti-particle of that particle (e.g. $\bar{\nu}$ for anti-neutrino) and it is recommended to use the same indication when needed in other cases.

5. Quantum States

1. A symbol indicating the quantum state of a *system* should be printed in capital roman (upright) type.

The right subscript indicates the total angular momentum quantum number and the left superscript the multiplicity.

Example: $^2\text{P}_1$ ($J = \frac{3}{2}$, multiplicity: 2)

2. A symbol indicating the quantum state of a single *particle* should be printed in lower case roman (upright) type.

The right subscript may be used to indicate the total angular momentum quantum number of the particle in the case of $j-j$ coupling.

Example: $p_{\frac{1}{2}}$ —electron

3. The letter symbols corresponding to the *angular momentum quantum number* should be:

0 S,s	4 G,g	8 L,l
1 P,p	5 H,h	9 M,m
2 D,d	6 I,i	10 N,n
3 F,f	7 K,k	11 O,o

6. Nomenclature

1. *Use of the word specific.*

The word 'specific' in English names for physical quantities should be restricted to the meaning 'divided by mass'.

Examples:

specific volume	volume/mass
specific energy	energy/mass
specific heat capacity	heat capacity/mass

2. *Notation for covariant character of coupling*

S	Scalar coupling	A	Axial vector coupling
V	Vector coupling	P	Pseudoscalar coupling
T	Tensor coupling		

3. *Abbreviated notation for a nuclear reaction.*

The meaning of the symbolic expression indicating a nuclear reaction should be the following:

$$\begin{array}{ccc} \text{initial} & \left(\begin{array}{cc} \text{incoming} & \text{outgoing} \\ \text{particle(s)} & \text{particle(s)} \end{array} \right) & \text{final} \\ \text{nuclide} & \left(\begin{array}{cc} \text{or quanta,} & \text{or quanta} \end{array} \right) & \text{nuclide} \end{array}$$

Examples:



4. *Character of transitions*

Multipolarity of transition:

electric or magnetic	monopole	E0 or M0	
"	"	dipole	E1 or M1
"	"	quadrupole	E2 or M2
"	"	octupole	E3 or M3
"	"	2 ⁿ -pole	E _n or M _n

parity change in transition:

transition with parity change:	yes
transition without parity change:	no

5. *Nuclide:* A species of *atoms*, identical as regards atomic number and mass number should be indicated by the word *nuclide*, not by the word *isotope*.

Different nuclides having the same atomic number should be indicated as *isotopes* or *isotopic nuclides*.

Different nuclides having the same mass number should be indicated as *isobars* or *isobaric nuclides*.

6. *Sign of polarization vector (Basel Convention)*

In nuclear interactions the positive polarization of particles with spin $\frac{1}{2}$ is taken in the direction of the vector product

$$\mathbf{k}_i \times \mathbf{k}_0,$$

where $\mathbf{k}_i$ and $\mathbf{k}_0$ are the circular wave vectors of the incoming and outgoing particles respectively.

7. Recommended Symbols for Physical Quantities

Remark:

(1) Where several symbols are given for one quantity, and no special indication is made, they are on equal footing.

(2) In general no special attention is paid to the name of the quantity.

7.1 Space and time

length	l
breadth	b
height	h
radius	r
diameter: $d = 2r$	d
path: $L = \int ds$	L, s
area	A, S
volume	V, v
plane angle	$\alpha, \beta, \gamma, \theta, \vartheta, \varphi$
solid angle	ω, Ω
wave length	λ
wave number: $\sigma = 1/\lambda$	$\sigma, \tilde{\nu}$
circular wave number: $k = 2\pi/\lambda$	k
time	t
period	T
frequency: $\nu = 1/T$	ν, f
angular frequency, pulsance: $\omega = 2\pi\nu$	ω
velocity: $v = ds/dt$	c, u, v
angular velocity: $\omega = d\varphi/dt$	ω
acceleration: $a = dv/dt$	a
angular acceleration: $\alpha = d\omega/dt$	α
gravitational acceleration	g
standard gravitational acceleration	g_0
v/c	β

7.2 Mechanics

mass	m
density: $\rho = m/V$	ρ
reduced mass	μ
momentum: $p = mv$	$p, \hat{p}$
moment of inertia: $I = \int r^2 dm$	I, J
force	$F, \mathbf{F}$
weight	$G, (W)$
moment of force	$M, \mathbf{M}$
pressure	p
normal stress	σ
shear stress	τ
gravitational constant:	
$F(r) = G m_1 m_2 / r^2$	G
modulus of elasticity, Young's modulus: $\sigma = E \Delta l / l$	E
shear modulus: $\tau = G \tan \gamma$	G
compressibility: $\kappa = - (1/V) dV/dp$	κ
bulk modulus: $K = 1/\kappa$	K
viscosity	η

kinematic viscosity: $\nu = \eta/\rho$	ν
friction coefficient	f
surface tension	γ, σ
energy	E, U
potential energy	V, E_p
kinetic energy	T, E_k
work	W, A
power	P
efficiency	η
Hamiltonian function	H
Lagrangian function	L
relative density	d

7.3 Molecular physics

number of molecules	N
number density of molecules: $n = N/V$	n
Avogadro's constant	L, N_A
molecular mass	m
molecular velocity vector with components	$\mathbf{c}, (c_x, c_y, c_z)$ $\mathbf{u}, (u_x, u_y, u_z)$
molecular position vector with components	$\mathbf{r}, (x, y, z)$
molecular momentum vector with components	$\mathbf{p}, (p_x, p_y, p_z)$
average velocity	$\mathbf{c}_0, \mathbf{u}_0, \bar{c}, \bar{u}$
most probable speed	$\bar{c}, \bar{u}$
mean free path	l
molecular attraction energy	ϵ
interaction energy between molecules i and j	φ_{ij}, V_{ij}
velocity distribution function: $n = \iint f d c_x d c_y d c_z$	$f(c)$
Boltzmann's function	H
generalized coordinate	q
generalized momentum	p
volume in γ phase space	Ω
thermodynamic temperature	$T, (\Theta)$
Boltzmann's constant	k
$1/kT$ in exponential functions	β
gas constant per mole	R
partition function	Q, Z
diffusion coefficient	D
thermal diffusion coefficient	D_T
thermal diffusion ratio	K_T
thermal diffusion factor	α_T
characteristic temperature	Θ
Debye temperature: $\Theta_D = h\nu_D/k$	Θ_D
Einstein temperature: $\Theta_E = h\nu_E/k$	Θ_E
rotational temperature: $\Theta_r = h^2/8\pi^2 I k$	Θ_r
vibrational temperature: $\Theta_v = h\nu/k$	Θ_v

7.4 Thermodynamics

quantity of heat	Q
work	W, A
temperature	$t, (\vartheta)$
thermodynamic temperature	$T, (\Theta)$

* $\tilde{\nu}$ is exclusively used in molecular spectroscopy.

** In physics: ν .

† Preferred symbol: G .

* Preferred symbol: T .

** Preferred symbol: t .

entropy	S
internal energy	U
Helmholtz function, free energy:	
$F = U - TS$	F
enthalpy: $H = U + pV$	H
Gibbs function: $G = U + pV - TS$	G
linear expansion coefficient	α
cubic expansion coefficient	γ
thermal conductivity	λ
specific heat capacity	c_p, c_v
molar heat capacity	C_p, C_v
Joule-Thomson coefficient	μ
ratio of specific heats	κ, γ

7.5 Electricity and magnetism

quantity of electricity	Q
charge density	ρ
surface charge density	σ
electric potential	V, Φ
electric field	E, E
electric displacement	D, D
capacitance	C
permittivity: $\epsilon = D/E$	ϵ
permittivity of vacuum	ϵ_0
relative permittivity: $\epsilon_r = \epsilon/\epsilon_0$	ϵ_r
dielectric polarization: $D = \epsilon_0 E + P$	P, P
electric susceptibility	χ_e
polarizability	α, γ
electric dipole moment	p, p
electric current	I
electric current density	J, J
magnetic field	H, H
magnetic induction	B, B
magnetic flux	Φ
permeability: $\mu = B/H$	μ
permeability of vacuum	μ_0
relative permeability: $\mu_r = \mu/\mu_0$	μ_r
magnetization: $B = \mu_0(H + M)$	M, M
magnetic susceptibility	χ_m
electromagnetic moment:	
(dimension: M/n)	$\mathbf{u}, \mu, m, m$
magnetic polarization: $B = \mu_0 H + J$	J
magnetic dipole moment:	
(dimension: J/n)	j, j
resistance	R
reactance	X
impedance: $Z = R + iX$	Z
admittance: $Y = 1/Z = G + iB$	Y
conductance	G
susceptance	B
resistivity	ρ
conductivity: $1/\rho$	γ, σ
self inductance	L
mutual inductance	M, L_{12}
phase number	m
loss angle	δ
number of turns	N
power	P

Poynting vector	S, S
vector potential	A

7.6 Light, radiation

quantity of light	Q
luminous flux	Φ
luminous intensity: $d\Phi/d\omega$	I
illumination: $d\Phi/dS$	E
luminance: $dI/dS \cos \theta$	L
luminous emittance: $d\Phi/dS$	M
quantity of radiant energy	Q_a
radiant flux	Φ_e, P
radiant intensity	I_e
irradiance	E_e
radiance	L_e
radiant emittance	M_e
absorption factor: Φ_a/Φ_0	α
reflection factor: Φ_r/Φ_0	ρ
transmission factor: Φ_{tr}/Φ_0	τ
absorption coefficient	a
extinction coefficient	κ
speed of light in empty space	c
refractive index: $n = c/c_n$	n

7.7 Acoustics

velocity of sound	c
velocity of longitudinal waves	c_l
velocity of transversal waves	c_t
group velocity	c_g
sound energy flux	P
reflection factor: P_r/P_0	ρ
acoustic absorption factor: $1 - \rho$	$\alpha_a, (\alpha)$
transmission factor: P_{tr}/P_0	τ
dissipation factor: $\alpha_a - \tau$	δ
loudness level	$L_N, (L)$

7.8 Atomic and nuclear physics

atomic number, proton number	Z
mass number	A
proton number: $P = Z$	P
neutron number: $N = A - Z$	N
charge of positron	e
electron mass	m, m_e
proton mass	m_p
neutron mass	m_n
meson mass	m_π, m_μ
nuclear mass	M_N, M
atomic mass	M_a, M
relative atomic mass: M_a/m_u	A_r
(unified) atomic mass constant:	
$m_u = M_a(^{12}\text{C})/12$	m_u
magnetic moment of particle	μ
magnetic moment of proton	μ_p
magnetic moment of neutron	μ_n
magnetic moment of electron	μ_e
Bohr magneton	μ_B, β
Planck constant	h
principal quantum number	n, n_i

* Written in rationalized, 4-dimensional form. See section 10.

orbital angular momentum quantum number	L, l_i
spin quantum number	S, s_i
total angular momentum quantum number	J, j_i
magnetic quantum number	M, m_i
nuclear spin quantum number	I
hyperfine quantum number	F
rotational quantum number	J, K
vibrational quantum number	v
quadrupole moment	Q
Rydberg constant	R_∞
Bohr radius: $a_0 = \hbar^2/m_e^2$	a_0
fine structure constant: $\alpha = e^2/\hbar c$	α
mass excess: $M_a - Am_a$	Δ
packing fraction: Δ/Am_a	f
nuclear radius: $R = r_0 A^{1/3}$	R
nuclear magneton	μ_N
g-factor: e.g. $g = \mu/I\mu_N$	g
gyromagnetic ratio: $\gamma = \mu/I\hbar = g(\mu_N/\hbar)$	γ
Larmor (angular) frequency	ω_L
level width	Γ
mean life	τ
reaction energy	Q
cross section	σ
macroscopic cross section: $\Sigma = n\sigma$	Σ
impact parameter	b
scattering angle	$\vartheta, \theta, \varphi$
internal conversion coefficient	α
disintegration energy	Q
half life	$T_{1/2}$
decay constant, disintegration constant	λ
activity	A
Compton wavelength: $\lambda_c = \hbar/mc$	λ_c
electron radius: $r_e = e^2/mc^2$	r_e
linear absorption coefficient	μ, μ_1
atomic absorption coefficient	μ_a
mass absorption coefficient	μ_m
linear stopping power	S, S_1
atomic stopping power	S_a
linear range	R, R_1
recombination coefficient	A

7.9 Chemical physics

amount of substance	ν, n
molar mass of substance B	M_B
molar concentration of subst. B	c_B
mole fraction of subst. B	x_B, X_B
mass fraction of subst. B	w_B
volume fraction of subst. B	φ_B
mole ratio of solution	r
molality of solution	m
chemical potential	μ
absolute activity of subst. B (dimensionless)	λ_B
relative activity	a_B

* n is used in chemistry, ν may be used as an alternative to n , when n is used for number density of particles.

activity coefficient	γ_B
osmotic pressure	Π
osmotic coefficient	g, φ
stoichiometric number of molec. B	ν_B
affinity	A
extent of reaction	ξ
equilibrium constant	K
charge number of ion	z
Faraday constant	F
ionic strength	I
activity of substance B (dimension of number density)	s_B

8. Recommended Mathematical Symbols

8.1 General symbols

equal to	$=$
not equal to	$\neq, \neq$
identically equal to	$\equiv$
corresponds to	$\hat{=}$
approximately equal to	$\approx$
proportional to	$\sim, \propto$
approaches	$\rightarrow$
larger than	$>$
smaller than	$<$
much larger than	$\gg$
much smaller than	$\ll$
larger than or equal to	$\geq, \geq, \geq$
smaller than or equal to	$\leq, \leq, \leq$
plus	$+$
minus	$-$
plus or minus	$\pm$
a multiplied by b	$ab, a \cdot b, a \cdot b, a \times b$
a divided by b	$a/b, \frac{a}{b}$
a raised to the power n	a^n
magnitude of a	$ a $
square root of a	$\sqrt{a}, \sqrt{a}, a^{1/2}$
mean value of a	$\bar{a}, \langle a \rangle$
factorial p	$p!$
binomial coefficient: $n!/p!(n-p)!$	$\binom{n}{p}$
infinity	∞

* **8.2 Letter symbols** and letter expressions for mathematical operations should be written in roman (or upright) type

exponential of x	$\exp x, e^x$
base of natural logarithms	e
logarithm to the base a of x	$\log_a x$
natural logarithm of x	$\ln x$
decadic logarithm of x	$\lg x, \log x$
binary logarithm of x	$\text{lb } x, \log_2 x$
summation	Σ
product	Π
finite increase of x	Δx

* In case of ambiguity $\log_{10} x$.

** Greek capital delta, not triangle.

variation of x	δx
total differential of x	dx
function of x	$f(x), f(x)$
limit of $f(x)$	$\lim f(x)$

8.3 Trigonometric functions

sine of x	$\sin x$
cosine of x	$\cos x$
tangent of x	$\tan x, \operatorname{tg} x$
cotangent of x	$\cot x, \operatorname{ctg} x$
secant of x	$\sec x$
cosecant of x	$\operatorname{cosec} x$

Remarks:

- a. It is recommended to use for the *inverse circular functions* the symbolic expressions for the corresponding circular function preceded by the letters: arc.

Examples: $\arcsin x$, $\arccos x$, $\arctan x$ or $\operatorname{arctg} x$, etc. Sometimes the notation $\sin^{-1} x$, $\tan^{-1} x$, etc. is used.

- b. It is recommended to use for the *hyperbolic functions* the symbolic expressions for the corresponding circular function, followed by the letter: h.

Examples: $\sinh x$, $\cosh x$, $\tanh x$ or $\operatorname{tgh} x$, etc.

- c. It is recommended to use for the *inverse hyperbolic functions* the symbolic expression for the corresponding hyperbolic function preceded by the letters: ar.

Examples: $\operatorname{arsinh} x$, $\operatorname{arcosh} x$, etc.

8.4 Complex quantities

imaginary unit ($i^2 = -1$)	i, j
real part of z	$\operatorname{Re} z, z'$
imaginary part of z	$\operatorname{Im} z, z''$
modulus of z	$ z $
argument of z : $z = z \exp i\varphi$	$\arg z, \varphi$
complex conjugate of z , conjugate of z	z^*

Remark: Sometimes the notation $\bar{z}$ is used for the complex conjugate of z .

8.5 Vector calculus. (see also 1. 2. 3)

absolute value	$ A , A$
differential vector operator	$\partial/\partial \mathbf{r}, \nabla$
gradient	$\operatorname{grad} \varphi, \nabla \varphi$
divergence	$\operatorname{div} A, \nabla \cdot A$
curl	$\operatorname{curl} A, \operatorname{rot} A, \nabla \times A$
Laplacian	$\Delta \varphi, \nabla^2 \varphi$
d'Alembertian	$\square \varphi$

8.6 Matrix calculus.

transpose of matrix A	$\bar{A}_{ij} = A_{ji}$
complex conjugate of A	$(A^*)_{ij} = (A_{ij})^*$
Hermitian conjugate of A	$(A^\dagger)_{ij} = A_{ji}^* = \bar{A}^\dagger$

9. International Symbols for Units

9.1 Unit systems

1. A *coherent system of units* is a system based on a certain set of "basic units," from which all "derived units" are obtained by multiplication or division without introducing numerical factors. In addition there are "dimensionless units," in particular the radian, symbol: rad, for plane angle and the steradian, symbol: sr, for solid angle.

2. The **CGS system** or cm-g-s system is a coherent system of units based on *three basic units* for the three basic quantities length, mass and time respectively:

centimetre	cm
gramme	g
second	s

In the field of *mechanics* the following units of this system have special names and symbols, which have been approved by the General Conference on Weights and Measures:

l, b, h	centimetre	cm
t	second	s
m	gramme	g
f, ν	hertz ($= s^{-1}$)	Hz
F	dyne ($= g \cdot cm/s^2$)	dyn
E, U, W, A	erg ($= g \cdot cm^2/s^2$)	erg
p	microbar ($= dyn/cm^2$)	μbar
η	poise ($= dyn \cdot s/cm^2$)	P

In the field of *electricity and magnetism* several variants of the CGS unit system have been developed, in particular the *electrostatic CGS system* and *electromagnetic CGS system*. Special names and symbols for some of the units of the second system are:*

H	oersted ($= cm^{1/2} \cdot g^{1/2} \cdot s^{-1}$)	Oe
B	gauss ($= cm^{1/2} \cdot g^{1/2} \cdot s^{-1}$)	G
Φ	maxwell ($= cm \cdot g^{1/2} \cdot s^{-1}$)	Mx

For further information about the units and unit systems in electricity and magnetism see section 10.

3. The **MKSA system** or m-kg-s-A system is a coherent system of units for mechanics, electricity and magnetism, based on *four basic units* for the four basic quantities length, mass, time and electric current intensity:

metre	m
kilogramme	kg
second	s
ampere	A

Remark: The system based on these four units was given the name *Giorgi system* by the International Electrotechnical Committee in 1958. The mechanical system, which is based on the first three units only, has the name *MKS system*.

* (See also 10.3.)

The following units of the MKSA system have special names and symbols, which have been approved by the General Conference on Weights and Measures:

l, b, h	metre	m
t	second	s
m	kilogramme	kg
ν, f	hertz ($= s^{-1}$)	Hz
F	newton ($= kg.m/s^2$)	N
E	joule ($= kg.m^2/s^2$)	J
P	watt ($= J/s$)	W
I	ampere	A
Q	coulomb ($= A.s$)	C
V	volt ($= W/A$)	V
C	farad ($= C/V$)	F
R	ohm ($= V/A$)	Ω
L	henry ($= Vs/A$)	H
Φ	weber ($= V.s$)	Wb
B	tesla ($= Wb/m^2$)	T

4. In the field of *thermodynamics* one introduces an additional basic unit, corresponding to the basic quantity:

thermodynamic temperature, the unit being the degree Kelvin, symbol: °K.

When the *customary temperature* is used, defined by $t = T - T_0$, where $T_0 = 273.15$ °K, this is usually expressed in degrees Celsius, symbol: °C. For *temperature interval* the name degree, symbol: deg, is often used, the indications "Kelvin" or "Celsius," indicating the zero-point of the temperature scale used, being irrelevant in this case.

5. In the field of *photometry* one introduces an additional basic unit, corresponding to the basic quantity: *luminous intensity*, this unit being the candela, symbol: cd.

Special names for units in this field are:

I	candela	cd
Φ	lumen	lm
E	lux ($= lm/m^2$)	lx

6. *The International System of Units*. For the system based on the six basic units:

metre	m	ampere	A
kilogramme	kg	degree Kelvin	°K
second	s	candela	cd

The name *International System of Units* has been recommended by the Conférence Générale des Poids et Mesures in 1960:

7. In the field of *chemical and molecular physics*, in addition to the basic quantities defined above having been defined by the Conférence Générale des Poids et Mesures, *amount of substance* is also treated as a basic quantity. The recommended basic unit is the mole, symbol: mol. The mole is defined as the amount of substance, which contains the same number of molecules (or ions, or atoms, or electrons, as the case may be), as there are atoms in exactly 12 grammes of the pure carbon nuclide ^{12}C .

9.2 Incoherent units

l	ångström	Å
σ	barn ($= 10^{-28} cm^2$)	b
V	litre	l
t, τ, T_1	minute	min
t, τ, T_1	hour	h
t, τ, T_1	day	d
t, τ, T_1	year	a
p	atmosphere	atm
P	kilowatt-hour	kWh
Q	calorie	cal
Q	kilocalorie	kcal
E, Q	electronvolt	eV
m	ton ($= 1000 kg$)	t
M_a, m	(unified) atomic mass unit	u
p	bar ($= 10^6 dyn/cm^2$)	bar

Remark: The (unified) atomic mass unit is defined as $\frac{1}{12}$ th of the mass of an atom of the ^{12}C nuclide.

APPENDIX.* Systems of Quantities and Units in Electricity and Magnetism

The CGS unit system with three basic units and the MKSA unit system with four basic units correspond respectively to two different sets of equations in the field of electricity and magnetism, which are developed starting from three and from four basic quantities respectively. These systems are denoted as three and four "dimensional" systems of equations respectively.

1. Systems of equations with 3 basic quantities

Three distinct sets of equations with three basic quantities** have been developed in the field of electricity and magnetism. These are:

(1.a) *The "electrostatic system" of equations*, defining the electric charge on the basis of Coulomb's law for the force between two electric charges, by taking the permittivity in vacuo equal to a dimensionless quantity, the number unity.

(1.b) *The "electromagnetic system" of equations*, defining the electric current on the basis of the interaction law for the force between two electric current elements, by taking the permeability in vacuo equal to a dimensionless quantity, the number unity.

(1.c) *The "symmetrical system" of equations*, using the electric quantities from system (1.a) and the magnetic

* The SUN commission, after reproducing in the previous chapters all the recommendations on symbols, units and nomenclature approved by the IUPAP, gives in this Appendix some factual information about existing systems of quantities and units in the field of electricity and magnetism.

** Often length, time and mass are chosen as basic quantities, but also other choices, e.g. length, time and energy or length, time and force have been used.

quantities from system (1.b). As a result of combining the two sets of quantities the velocity of light in vacuo appears explicitly in some of the equations interrelating electric and magnetic quantities.

The equations in these three systems are usually written in the "nonrationalized" form, which is called "non-rationalized," because in these equations often factors 2π or 4π appear in situations not involving circular or spherical symmetry respectively. These equations are sometimes written in a "rationalized" form, in which these factors appear only in those equations, where they could be expected from the geometry of the system. However this rationalized form is only rarely used. The nonrationalized symmetrical system of equations (1.c) is commonly used in theoretical physics.

2. Systems of equations with 4 basic quantities

In the equations with four basic quantities at least one quantity of electric or magnetic nature is included in the basic set. In such a system the permittivity and the permeability in vacuo appear explicitly as physical quantities in the relevant equations.

Two different sets of equations are in use:

(2.a) The "nonrationalized system" of equations, in which the factors 4π and 2π often appear at unexpected places.

(2.b) The "rationalized system" of equations, in which these factors only appear in those equations, where they could be expected from the geometry.

When four-dimensional equations are used, one commonly writes these equations in the rationalized form (2.b).

Some characteristic equations of the three-dimensional "symmetric system" (1.c) and the corresponding four-dimensional equations in the nonrationalized form (2.a) and the rationalized form (2.b) are given in the table. The

quantities of the three-dimensional "symmetric system" of equations (1.c) have been indicated with an asterisk (*), those of the nonrationalized four-dimensional equations (2.a) with a prime ('), as far as they are different from those of the system (2.b).

Remark on the rationalization:

The basis of these considerations in which the rationalization is connected with the writing of the equations between physical quantities is in agreement with the resolution accepted by the IUPAP in 1951 in Copenhagen:

"The General Assembly of the Union of Physics considers that, in the case that the equations are rationalized, the rationalization should be effected by the introduction of new quantities."

("L'Assemblée Générale de l'Union de Physique considère que, dans le cas où les équations sont rationalisées, la rationalisation doit être obtenue par l'introduction de grandeurs nouvelles.")

3. CGS system of units

(3.a) The *electrostatic CGS system of units* forms a coherent system of units in combination with the three-dimensional "electrostatic system" of equations (1.a).

(3.b) The *electromagnetic CGS system of units* forms a coherent system of units in combination with the three-dimensional "electromagnetic system" of equations (1.b).

(3.c) The *mixed CGS system of units*, consisting of the set of electrostatic units of the electrostatic CGS system on the one hand and the set of magnetic units of the electromagnetic CGS system on the other hand, form together also a coherent system of units, when used in combination with the three-dimensional "symmetrical system" of equations (1.c).

<i>Symmetrical equations with 3 basic quantities</i>	<i>Nonrationalized eq. with 4 basic quantities</i>	<i>Rationalized eq. with 4 basic quantities</i>
$c \operatorname{rot} \mathbf{E}^* = -\partial \mathbf{B}^* / \partial t$	$\operatorname{rot} \mathbf{E} = -\partial \mathbf{B} / \partial t$	$\operatorname{rot} \mathbf{E} = -\partial \mathbf{B} / \partial t$
$\operatorname{div} \mathbf{D}^* = 4\pi \rho$	$\operatorname{div} \mathbf{D}' = 4\pi \rho$	$\operatorname{div} \mathbf{D} = \rho$
$\operatorname{div} \mathbf{B}^* = 0$	$\operatorname{div} \mathbf{B} = 0$	$\operatorname{div} \mathbf{B} = 0$
$c \operatorname{rot} \mathbf{H}^* = 4\pi \mathbf{i}^* + \partial \mathbf{D}^* / \partial t$	$\operatorname{rot} \mathbf{H}' = 4\pi \mathbf{i} + \partial \mathbf{D}' / \partial t$	$\operatorname{rot} \mathbf{H} = \mathbf{i} + \partial \mathbf{D} / \partial t$
$\mathbf{F} = Q^* \mathbf{E}^* + Q^* \mathbf{v} \times \mathbf{B}^* / c$	$\mathbf{F} = QE + Q\mathbf{v} \times \mathbf{B}$	$\mathbf{F} = QE + Q\mathbf{v} \times \mathbf{B}$
$w = (\mathbf{E}^* \cdot \mathbf{D}^* + \mathbf{B}^* \cdot \mathbf{H}^*) / 8\pi$	$w = (\mathbf{E} \cdot \mathbf{D}' + \mathbf{B} \cdot \mathbf{H}') / 8\pi$	$w = (\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}) / 2$
$S = c(\mathbf{E}^* \times \mathbf{H}^*) / 4\pi$	$S = (\mathbf{E} \times \mathbf{H}') / 4\pi$	$S = (\mathbf{E} \times \mathbf{H})$
$\mathbf{E}^* = -\operatorname{grad} V^* - (1/c) \partial \mathbf{A}^* / \partial t$	$\mathbf{E} = -\operatorname{grad} V - \partial \mathbf{A} / \partial t$	$\mathbf{E} = -\operatorname{grad} V - \partial \mathbf{A} / \partial t$
$\mathbf{B}^* = \operatorname{rot} \mathbf{A}^*$	$\mathbf{B} = \operatorname{rot} \mathbf{A}$	$\mathbf{B} = \operatorname{rot} \mathbf{A}$
$\epsilon_r \mathbf{E}^* = \mathbf{D}^*$	$\epsilon_0' \epsilon_r \mathbf{E} = \epsilon' \mathbf{E} = \mathbf{D}'$	$\epsilon_0 \epsilon_r \mathbf{E} = \epsilon \mathbf{E} = \mathbf{D}$
$\mathbf{E}^* = \mathbf{D}^* - 4\pi \mathbf{P}^*$	$\epsilon_0' \mathbf{E} = \mathbf{D}' - 4\pi \mathbf{P}$	$\epsilon_0 \mathbf{E} = \mathbf{D} - \mathbf{P}$
$\mathbf{B}^* = \mu_r \mathbf{H}^*$	$\mathbf{B} = \mu' \mathbf{H}' = \mu_0' \mu_r \mathbf{H}'$	$\mathbf{B} = \mu \mathbf{H} = \mu_0 \mu_r \mathbf{H}$
$\mathbf{B}^* = \mathbf{H}^* + 4\pi \mathbf{M}^*$	$\mathbf{B} = \mu_0' (\mathbf{H}' + 4\pi \mathbf{M})$	$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$

In physics this mixed or symmetrical CGS system of units combined with the three-dimensional "symmetrical system" of equations, is commonly used.

4. MKSA system of units

The MKSA system of units forms a coherent system of units in either of the four-dimensional systems of equations mentioned under 2.

The MKSA system is, however, most commonly used together with the rationalized (four-dimensional) equations (2.b).

5. centimetre-gramme-second-franklin system and centimetre-gramme-second-biot system

Several investigators have pointed out over many years the advantages, which result from the use of the four-dimensional equations, i.e. equations with four basic quantities of which at least one is of electrical nature. These advantages are partly of a didactical nature, but many investigators consider the usage of four basic quantities also important for developing a clear representation of the field of electricity and magnetism.

It has often been considered as a disadvantage that the transition from the three- to the four-dimensional system of equations should be tied to the transition from the CGS system of units to the MKSA system of units as explained under 3, and 4, which as a consequence would lead to a change in the numerical value of many well known physical quantities.

For that reason several investigators have advocated the introduction of a four-dimensional system of units, which is a generalization of the CGS system. This unit-system is chosen so that to any unit of the CGS system (with three basic units) there corresponds one particular unit of this "generalized CGS system" (with four basic units), such that the numerical values of the physical quantities in the field of electricity are invariant.

The introduction of such a "generalized CGS system" has the advantage that the relation between the units of this "generalized CGS system" and the units of the MKSA system can be expressed by ordinary conversion relations.

The use of the four-dimensional system of equations (with four basic quantities) accompanied by the usage of such a "generalized CGS system" of units (with four basic units) is therefore of advantage as an intermediate representation in this period of coexistence of the CGS system and the MKSA system, where transitions from one system to the other often have to be made.

As a result of all these considerations the IUPAP in the General Assembly of the IUPAP, Copenhagen, 1951, resolution 5, approved the introduction of the following two "generalized CGS systems" based on four basic units:

(5.a) The system with the *centimetre, gramme, second* and the *electrostatic unit of charge*, as basic units.

(5.b) The system with the *centimetre, gramme, second* and the *electromagnetic unit of current*, or deca ampere, as basic units.

In practical applications of these systems it is of advantage to indicate the *basic electrical unit* in each of these two systems with a name and symbol.

The name franklin was proposed in 1941 for the electrostatic unit of electric charge considered as a basic unit of the system (5.a). The franklin, symbol: Fr, is thus defined as:

The *franklin* is that charge, which exerts on an equal charge at a distance of 1 centimetre in vacuo a force of 1 dyne.

According to this definition 1 franklin = $(10/\xi)$ coulomb, where $\xi = 2,99793 \times 10^{10}$ is the numerical value of the velocity of light in vacuum, measured in cm/s.

For the electromagnetic unit of electric current, considered as a basic unit of the system (5.b), the name biot has been used. The biot, symbol: Bi, is thus defined as:

The *biot* is that constant current intensity, which, when maintained in two parallel rectilinear conductors of infinite length and of negligible circular section, placed at a mutual distance of 1 centimetre apart in vacuo, would produce between these conductors a force of 2 dyne per centimetre length.

According to this definition 1 biot = 10 ampere†.

These unit systems (5.a) and (5.b) are referred to as cm-g-s-Fr system and cm-g-s-Bi system respectively.

The definitions of the franklin in the system (5.a) and that of the biot in the system (5.b) which closely correspond to the electrostatic CGS unit of charge and the electromagnetic CGS unit of current respectively, ensure that the quantities of the *nonrationalized* four-dimensional system of equations (second column, table p. 29), when expressed in these units, have the same numerical value as the corresponding quantities of the nonrationalized three-dimensional system of equations (first column, table p. 29), when expressed in the corresponding CGS units.

Some of the units of the three-dimensional CGS system and the corresponding units of the two four-dimensional systems are given in the table:

three-dimensional CGS unit		corresponding four-dimensional unit	
Q^*	erg ^{1/2} . cm ^{1/2}	Q	Fr
V^*	erg ^{1/2} . cm ^{-1/2}	V	erg/Fr
E^*	erg ^{1/2} . cm ^{-3/2}	E	dyn/Fr
D^*	erg ^{1/2} . cm ^{-1/2}	D'	Fr/cm ²
C^*	cm	C	Fr ² /erg
ϵ_r	1	ϵ'	Fr ² /erg.cm
I^*	dyn ^{1/2}	I	Bi
B^*	G	B	dyn/Bi.cm
H^*	Oe	H'	Bi/cm
μ_r	1	μ'	dyn/Bi ²

† The wording of this definition is, except for the replacement of 1 metre by 1 centimetre and of 2 newton per metre by 2 dyne per centimetre, the exact translation of the original french text defining the ampere (C.R. 9me Conf. Gen. Poids et Mesures, 1948, p. 49).