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development of PLASMA PHYSICS in the LAST 10 YEARS

By ***W. P. Allis***

THIS article will review rapidly the development of plasma physics from the beginning. There is not very much to say about the time before 1900. There was essentially no theory of electricity. To be sure, there was a one-fluid theory propounded by Franklin and a two-fluid theory propounded by du Fay, but neither of these "theories" could predict anything quantitative. Therefore, they do not qualify fully as theories. They were merely pictures, and it did not matter which theory was held. The same conclusions were reached either way.

The limitation which prevented the development of plasma physics in early times had nothing to do with knowledge of electricity; it was partly the lack of continuous sources of electricity, but mainly it was that the vacuum pumps were not good enough. The gas pressures could not be brought down conveniently, and little can be done with discharges at atmospheric pressures without milli-microsecond timing, and that was not available. But as pumps got better, the scale of plasma phenomena increased, and Faraday was then able to observe the dark space named after him. He then observed the negative glow, but he could not distinguish the Crookes dark space. Crookes had a better pump than Faraday had, and he observed that dark space. As pumps got better yet, the "emanations" that came from the cathode were observed, and it was seen that these travel in straight lines, and that obstacles cast a shadow.

I believe this was the first precise observation in gas discharges. It immediately overthrew the fluid theories, because one expects a fluid to go around an obstacle, and the "emanations" were immediately named "rays", and people thought of them as caused by particles. The charge of the particle was already known because Faraday's constant and Avogadro's number were already known, and hence one knew that electricity had the same atomicity as atoms, and the charge was known. But the first measurement—quantitative measurement—was made in 1897 by J. J. Thomson, who measured e/m for cathode rays and observed that it was the same no matter what gas was used, what pressure was used, or what cathode was used, and hence, if e over m was constant, and if one knew e, one could calculate m, which turned out to be very small, but quite finite; thus the electron was "discovered", and that begins the quantitative theory of discharges in gases.

1897–1914: Townsend and the Free-Path Model

After J. J. Thomson, a theory existed, the electron theory, and from then on experiment was guided, or could be guided, by a theory which could suggest experiments, which could in turn suggest calculations. Developments came very rapidly in this era (from 1900 to 1913), which was dominated by J. S. Townsend.

The first quantitative observations by Townsend were related to breakdown. He devised a very ingenious, very simple little apparatus which is still being used (generally gold-plated nowadays), which consists of parallel plates mounted on tubes, one of which has a screw thread on it, as shown in Fig. 1. The thread is very important. The experimentally difficult thing to do is to change the length of the discharge without changing E/p. This is done by changing the position of one electrode by screwing it back, and changing the applied voltage proportionately. Electrons, liberated at one end

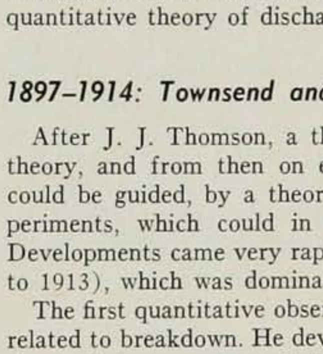

Fig. 1. Townsend's avalanche apparatus.

by ultraviolet light, produce an avalanche of electrons in the gas. This is the Townsend Avalanche.

What Townsend first suspected from theory and then proved by experiment was that the multiplication in the avalanche involves a constant exponential increase rate, a quantity alpha, and that alpha divided by the pressure is some function of the electric field divided by the pressure. So here we have the first formula

$$\alpha/p = f(E/p) \approx Ae^{Bp/E}.$$

This is the first equation in electrical discharges.

Townsend also had another piece of apparatus, shown in Fig. 2, in which he measured the diffusion, D, and the mobility, μ, of electrons or ions coming down through a hole. The electron stream falls on a series of plates, from which he can measure the currents, and this defines the spread of the stream, and the angle of spread of the stream measures the ratio D/μ. Townsend very carefully adjusted the electric field by plates at the sides connected through a drop wire, thus avoiding the errors due to wall charges. Then he applied a transverse magnetic field, which deflected the stream, and a measurement of the deflection and spread gave both D and μ, separately. Theory shows that $D/\mu = kT/e$ so that the temperature T can be determined. It was found that the ions are at gas temperatures, but the electrons are hotter. Townsend's theory is not adequate to explain this.

Townsend also developed another theoretical method —the mean-free-path method—which was used by him and by many of his followers and is still used now, largely in Australia. In this method it is assumed that all electrons have the same free path, l. This method is ingenious but has its troubles. For instance, there is a formula for the diffusion coefficient according to the mean-free-path method:

$$D = l\bar{v}/3,$$

where $\bar{v}$ is the average speed. The correct formula requires a bar over both l and v: $D = \overline{lv}/3$.

The product should be averaged. The characteristic of the mean-free-path method, is that the mean free path is treated as a constant, and this results in interesting errors. To illustrate further, there is a formula for mobility, $\mu = el/mv_{\mathrm{rms}}$.

We are careful this time; we take the rms velocity. This formula turns out to be not quite right, so one does a great deal about averaging angles of scattering to get the proper constant, and finds that

$$\mu = 0.92\ el/mv_{\mathrm{rms}}.$$

There are a great many papers on the details of the calculations, and the number is sometimes given as 0.85. However, nobody worries about the "l" in the mean-free-path theory, and it turns out that this may vary by factors of five, according to conditions. All the papers about the constant in the formula are rather irrelevant.

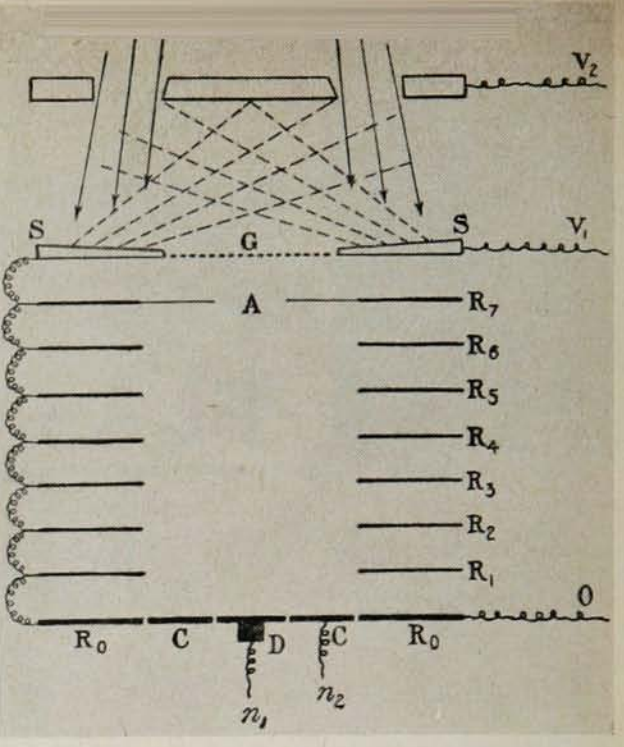

Fig. 2

There is a better theory that was available at the same time (from 1903 to 1914) that this work by Townsend was going on. In 1905, Lorentz had applied Boltzmann theory, not to electrical discharges, but to electrons in metals, and one can compare the formula above for the mobility to the correct one given by Boltzmann theory,

$$\mu = \int \frac{el}{3}\frac{\partial f}{\partial v} 4\pi v^2 dv.$$

Dimensionally, one sees that this is the same, but the average is over the derivative of f, and one has to know the distribution function f. However, at the time everybody was certain that distribution functions were Maxwellian. Indeed, Lorentz used this in his theory of metals, but nobody else used Boltzmann's theory at that time.

It is an interesting fact that all this work on electrical discharges was done before very much was known about atoms. The existence of energy gaps between sharp excited levels was unknown at this time. The work of Franck and Hertz, who first experimentally demonstrated the existence of energy levels in atoms by electrical discharge means, was done in 1913. It is rather amazing how much one can do, starting with ionization, which was the first result, without knowing ionization potentials, without knowing energy levels, without really knowing that the atom was anything other than the rigid sphere of ancient theory. This theory and method continued in use until World War I.

1918–1930: Langmuir and Space-Charge Theory

After the war we start a new era dominated by Langmuir and the theory of space charge.

Townsend could not understand active discharges. He did all his experiments on pre-breakdown or "dark" discharges in which the primary electrons were produced, not by the discharge itself, but externally by the photoelectric effect or by some other effect. He understood the avalanche, but not the "plasma", as it was to be named. The study of space charge started earlier than this. It started in 1911 with thermionic emission and Child who discovered the law which is now known as the Child-Langmuir law:

$$J = \frac{4\epsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{V^{3/2}}{X^2}.$$

But nothing came of it until 1913 when Langmuir rediscovered it. This law was first derived to explain observed currents in thermionic diodes. It had nothing to do with electrical discharges of the Townsend type.

After the war, certain mercury-pool discharges such as the Cooper-Hewitt light and the arc rectifier had reached industrial importance. Townsend had worked with air or rare gases at millimeter pressures, in which electron mean free paths were of the order of a millimeter. The mercury discharge operated at micron pressures with electron free paths of the order of a meter. Townsend's theories were inapplicable to these discharges. From 1923 to 1929, Langmuir and co-workers, Mott-Smith, Blodgett, and Tonks, developed the theory of electrical discharges with space charge. The first thing they did was to develop the Langmuir probe, and we have a sketch of the Langmuir probe in Fig. 3. There are many designs for such probes, and this is a wall probe. Just as a hot wire is separated from any collecting surface by a space-charge region, so Langmuir pictured the discharge to be separated from any walls or probes by a space-charge sheath. Everyone before Langmuir who introduced wires into discharges and measured their voltage thought that he measured the voltage at that point of space. It generally turned out to be strongly negative. By adopting a picture in which the discharge is separated from all material objects by a sheath, Langmuir was able to distinguish "floating potential" from "space potential" to show that the space potential, far from being negative, was usually positive, relative to the walls. In the sheath, there is a positive ion current coming out of the plasma. Langmuir wrote for this positive ion current,

$$J = ne\sqrt{kT/2\pi m}.$$

This is the standard formula for the random currents in a gas, and combining this with the Child formula, he obtained an equation for the thickness, s, of the space-charge "sheath":

$$s^2 = \frac{8\epsilon_0}{9n}\sqrt{\frac{\pi}{kTe}}\,V^{3/2} = \left(\frac{\pi\lambda_d}{3}\right)^2\left(\frac{4eV}{\pi kT}\right)^{3/2},$$

where λ_d is the Debye length

$$\lambda_d^2 = \epsilon_0 kT/ne^2.$$

I bring in the names of Debye and Hückel because they discovered the Debye length in 1923 in connection with the theory of electrolytes. If you write Langmuir's formula in terms of λ_d, you see that the sheath thickness is in general, a small multiple of the Debye length, because sheath voltages are a few times kT/e. So sheath thicknesses and Debye lengths are more or less the same thing.

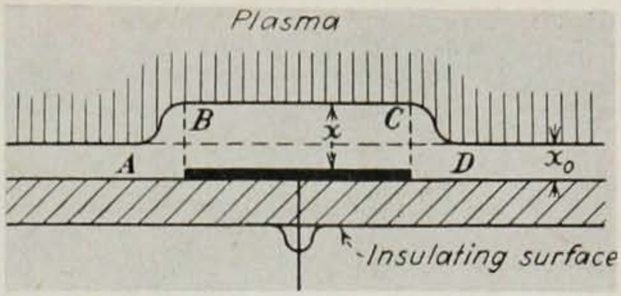

Fig. 3. Space-charge sheath on finite plane probe.

It was in 1929 that Langmuir and Tonks introduced the word "plasma" to distinguish that part of an active discharge which is not a sheath. In the sheath, the densities of the two kinds of particles are so different that one of them can be neglected, at least in first order. The other type of particle gives rise to space charge, which produces a large electric field, which in turn drives a current large compared to the local random current. In the plasma, the two kinds of particles are in practically equal numbers so that space charge can, in first order, be neglected. However, the currents, small compared to random currents, which flow in the plasma, require electric fields to drive them, and the divergence of the field determines a small difference in the densities of the two kinds of particles. The distinguishing characteristics of sheaths and plasmas may be summarized in the table:

Sheaths	*Plasmas*
$n_+ > 2n_-$	$n_+ - n_- < n_-$
$J > J_{\text{random}}$	$J < J_{\text{random}}$
$E = \frac{e}{\epsilon_0}\int (n_+ - n_-)dn$	$n_+ - n_- = \frac{\epsilon_0}{e}\nabla\cdot\mathbf{E}$

In sheaths, the last equation is essential. In plasmas, one need never use Poisson's equation, because it only serves to give the negligible difference in electron and ion concentrations.

In the year 1929, Langmuir's articles on discharges occupied a hundred pages of *The Physical Review*, and besides that, he had articles in other journals. In that year, he explained Langmuir's paradox. This paradox applies to a hot cathode placed in a low-pressure discharge, so that there is a double sheath around the hot cathode. The electrons fall through this sheath and form a beam. As has been said, the plasmas with which Langmuir worked were very-low-pressure ones with long mean free paths. However, he observed that the fast electrons from the cathode did not go across the tube, as the mean-free-path theory would have suggested. Langmuir observed in 1925 that they became diffused in a very much smaller distance, and this was the paradox. He now found that the answer lay in the excitation of plasma oscillations, by the passage of the fast electrons through the plasma. Langmuir and Tonks wrote the formula for the plasma oscillation frequency

$$\omega_p^2 = ne^2/\epsilon_0 m,$$

except, of course, he did not write in this way because the mks units which are adopted here were not used by Langmuir. This is not a wave, it is an oscillation. The positive-ion oscillations, however, propagate as a wave, and Langmuir and Tonks wrote the formula for its phase velocity. Langmuir and Tonks also studied sheaths further than they had before. Besides the single sheath in which one kind of particle, say electrons, are going to the right and giving a 3/2-power law for the voltage, if, simultaneously, ions are going to the left, the potential distribution becomes s-shaped, and this is called a double sheath; Langmuir showed that although the masses are far different, this is a symmetric curve, but that it can have horizontal tangents at both ends only when

$$I_+/I_- = \sqrt{m_-/m_+}.$$

Double sheaths of this kind exist at the surface of hot cathodes in a discharge and across the openings of grids.

At the end of this period, someone finally caught up with what should have been done much earlier, but was not. In 1930, Druyvesteyn first wrote down a distribution function which is not Maxwellian. This discovery was quite out of its period and should have come before 1913. It probably came at this time because Langmuir probes were exhibiting distributions which were non-Maxwellian for reasons quite different from those considered by Druyvesteyn.

1930–1950: Large Plasmas in Magnetic Fields

Plasmas which were studied for fifty years as laboratory curiosities are now discovered at every-increasing distances from the earth. Although the magnetic fields are generally low in these plasmas, collision frequencies are also low, even lower than the ion cyclotron frequency, and for that reason magnetic fields become important. The subject was introduced in 1928, when Appleton used echo-ranging for the ionosphere. Radiation of frequency lower than the plasma frequency will not penetrate the ionosphere, so that a radio pulse of suitable frequency is reflected, and the elapsed time for the pulse to return gives the height of the ionosphere—in principle, a simple way to ascertain the density of the ionosphere with height. Magnetic fields alter the reflection frequency, so in 1931 Hartree introduced magnetic fields into the theory, and from then on one studied the ionosphere in detail with radar pulsing.

But flight from the earth does not stop with the ionosphere. In 1937, Alfvén suggested that there are magnetic fields everywhere in space. He concluded this from measurements of polarization of light from gaseous nebulae, which are very good plasmas. Then Ferraro showed that a highly conducting medium is linked to the magnetic lines of force, so that they must move together. The proof is simple. The current, **J**, in a medium of conductivity σ moving with the velocity **V** through a magnetic field **B**, is:

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}).$$

If the current is finite but the conductivity is large, the bracket must be small. This yields a velocity $\mathbf{V} = \mathbf{E} \times \mathbf{B}/B^2$ which is precisely the velocity at which lines of magnetic force are assumed to move.

We saw how the First World War interrupted the development of electrical discharges. The Second World War did the same, except for astronomers. In 1942, Alfvén introduced the science of magnetohydrodynamics, or MHD for short, to explain sunspots. I shall not go into the explanation of sunspots, but as MHD is a popular topic at present, I would like to define it more precisely. We start by writing Maxwell's equations without Poisson's equation because, as we have seen, Poisson's equation is superfluous in a plasma.

$$\nabla \times \mathbf{H} = \mathbf{J} + \dot{\mathbf{D}}$$

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}$$

The displacement current $\dot{\mathbf{D}}$ was introduced by Maxwell and creates electrodynamics. It has no place in hydrodynamics because $\mathbf{J} \gg \dot{\mathbf{D}}$ and we drop it.

We then write Ohm's law,

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}),$$

and go to the Ferraro limit, in which $\mathbf{E} = \mathbf{B} \times \mathbf{V}$, and **J** is indeterminate. If the conductivity has a finite real part, one has a lossy MHD. We then write Newton's law,

$$-\nabla p + q\mathbf{E} + \mathbf{J} \times \mathbf{B} = \rho\dot{\mathbf{V}}.$$

On the left the three terms define compressive, electro- and magneto-theories. Alfvén assumed the $\mathbf{J} \times \mathbf{B}$ term to dominate and hence had a magneto-theory, but the compressible magneto-theory is also worked out. On the right the use of single density ρ marks it as a hydro-theory. Actually, there are at least two kinds of particles, and we need a two-fluid theory (shades of du Fay), but if the conductivity due to the ions alone is large, the two fluids are both tied to the lines of force and move together, and we can get away with a one-fluid hydro-theory. Finally, $\dot{\mathbf{V}}$ is obviously a dynamic term so that we have magnetohydrodynamics.

1950: Applications

Since 1950, magnetohydrodynamics has come down to earth, and we are in a period of applications: plasma jets, plasma propulsion, MHD generators, thermonuclear reactors, etc. These are too close to us to view with detachment and cannot yet form part of a history.