# OPTICS

## and INFORMATION THEORY

*By Richard K. McDonald*

*Modern optical technology is being revolutionized through the use of the statistical methods of information theory in problems involving light-gathering processes. The author is a preliminary designer of infrared and optical systems with the Boeing Airplane Company in Seattle, Washington.*

PHYSICAL processes can sometimes be best described by analogies to apparently unrelated situations. Thus, viscous flow of molasses might be visualized by a mechanical model of springs, dead weights, and dashpots, and the problems of pumping molasses through pipes might be simulated by the flow of electricity through resistors. One resorts to such physical analogies to take advantage of simpler mathematical procedures or more advanced theories in some other domain.

Few physical sciences are as ancient or as well understood as optics, yet within the past decade a significant change of emphasis has evolved. The designer now speaks of his light-gathering system in terms of electrical network theory and uses mathematical concepts long employed in the analysis of communication circuits. Basically, the modern innovation of information theory in optics is that one can now define how much information is present in a message, which can be an optical image, and then can describe the message on a statistical basis.

Most persons consider optical devices as inseparable from a human observer. Except for focusing and identification operations, modern military and scientific apparatus virtually preclude any human intervention. Even in astronomy, serious work for the past fifty years has largely depended on the photocell and the photographic plate. On the contemporary scene, most of the efforts of the optical industry are directed toward complex equipments using photographic film, television tubes, or some sort of photoelectric detector that registers photons by a change in electrical properties. To name representative systems, there are infrared guided missiles, automatic star trackers, infrared trackers, spectrophotometers, television cameras, aerial cameras, infrared gun sights, and satellite horizon sensors.

With such equipment, where the lens operates in the dimension of space and the electrical detector functions on a time basis, one finds the most successful applications of information theory. For the first time optical systems can be expressed in terms the electronic designer can use.

All those factors which influence the interpretation of optical intelligence can be thought of as appearing in a series sequence. Surprisingly, when we equate optical functions with the language of communication theory, close counterparts to the elements of an electrical circuit appear. The first is the entrance aperture, which acts much as a low band-pass electrical filter. The second would be the objective lens itself, whose function is comparable to a wide band-pass electrical amplifier. A field lens can be thought of as an electrical integrator, and a chopping reticle has the properties of a narrow band-pass filter (Fig. 1). These terms will be clarified later.

Suggesting that lenses behave as electrical filters

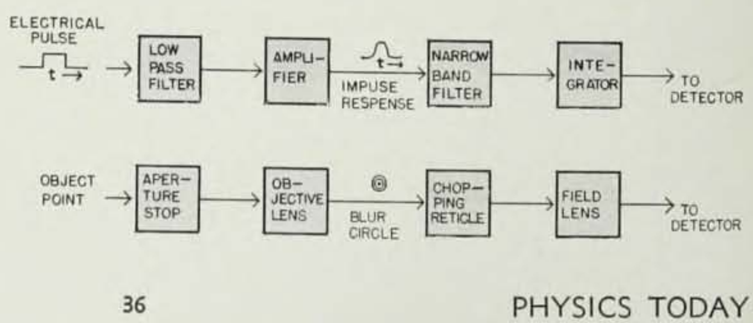


*Fig. 1.* Optical-electronic analog.

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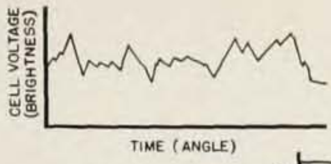
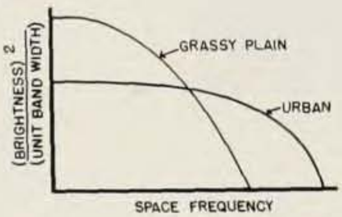



*Fig. 2.* Methods of representing optical intelligence: (a) point-by-point — photographic, (b) angle brightness, and (c) Wiener spectrum.

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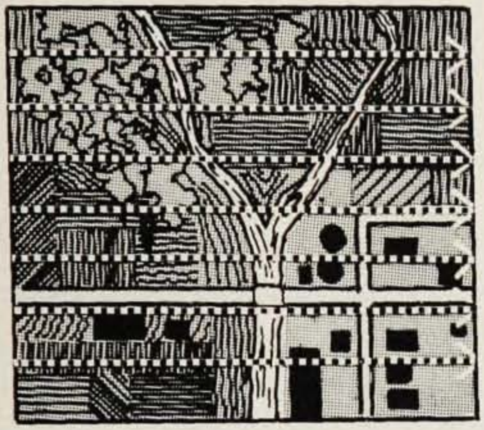


2a

implies that their characteristics depend on some frequency function. In optics, the frequency domain is not time frequency, but space frequency. We can best define "space frequency" by a demonstration of how information theory treats the contents of a pictorial scene.

Aside from a literal point-by-point description of the scene, as a photograph would do, the scene can be represented as the electrical output of a tiny photocell scanning back and forth, just the way an electron beam paints the lines on a television screen (Fig. 2). By this transformation, the three-dimensional information in the scene has been reduced to a two-dimensional signal which looks very much like electrical noise. Actually the voltage amplitude is directly proportional to light intensity, and the base axis of time corresponds to angular coordinates on the scene.

This signal can't be immediately interpreted by a visual observer, but for preserving data it overcomes many limitations of photographic film. It accommodates a wider range of brightness values than does film, is easier to calibrate, and can be readily processed to emphasize certain features of the original view. The method's principal disadvantage is apparent: it would take an enormous amount of tape to preserve as much information as can a high-quality photograph. A single 9.5-inch by 9.5-inch photograph having a resolving power of 1000 lines per millimeter has as many data bits as 10 000 hours of high-fidelity audio tape.

There is still a third method of describing optical objects which involves the Fourier transform. With a Fourier transform one can mathematically transpose most functions, whether they be a heat triangular wave form or chaotic noise, into the sum of a series of sine and cosine waves. Each member of the series will have a different frequency and amplitude. Electrical wave trains form the most graphic means of picturing Fourier transforms. These periodic functions will have an amplitude (voltage) that changes with time. Experimentally, it is easy to measure the constants of each of the harmonic sine waves that, taken all together, would recreate the original wave train.

We call this equivalent wave distribution the Fourier spectrum. A more useful quantity for electrical engineers is the power represented in each harmonic wave. This power is merely the square of the amplitude at each frequency, and the total distribution is designated as the power spectrum.

Until World War II the treatment of a physical function on a statistical basis was largely limited to electrical engineering fields. In 1948, publications by Norbert Wiener and Claude Shannon showed how the communication of any type of information could be regarded as a problem in statistics. Because of Wiener's work, the power spectrum as applied to the communication realm is called the Wiener spectrum. An enormous amount of data is represented by one Wiener spectrum. Furthermore, it offers the advantages of a wide dynamic range and the capability to preserve minute detail.

We derive the Wiener spectrum from a scene by calibrating the outputs of the small scanning photocell previously mentioned. Generally the units are brightness squared versus space frequency, expressed as waves per radian (one radian equals 57 degrees). There are both one- and two-dimensional Wiener spectra, depending on how the data were gathered.

One limitation of the technique is that one will have no idea of the shape of the original object, although different backgrounds will have dissimilar Wiener spectra. A featureless grassy plain will have far less high-frequency content than a city panorama with many sharp boundaries (Fig. 2c).

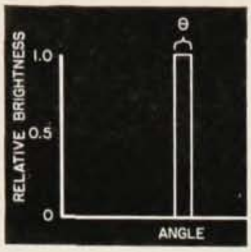



*Fig. 3a.* Angular distribution of energy resulting from a small object, size *θ*, surrounded by a uniform background.

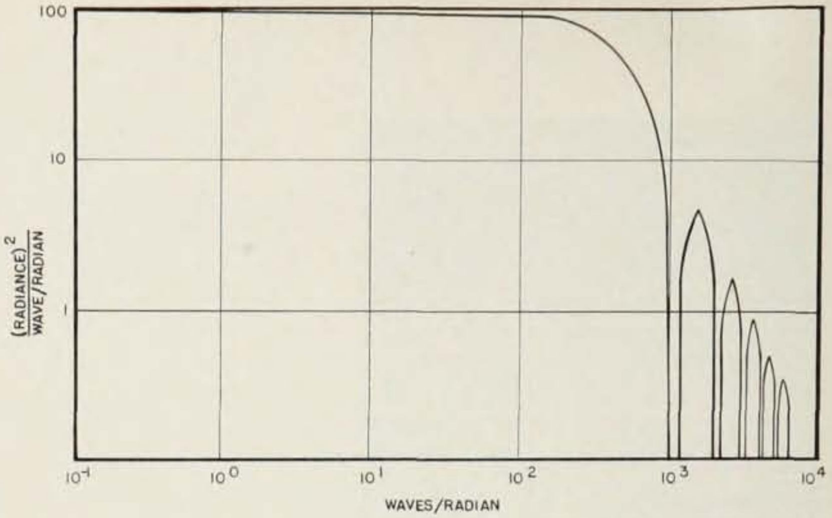



*Fig. 3b.* Wiener spectrum, point source ($10^{-3}$ radian).

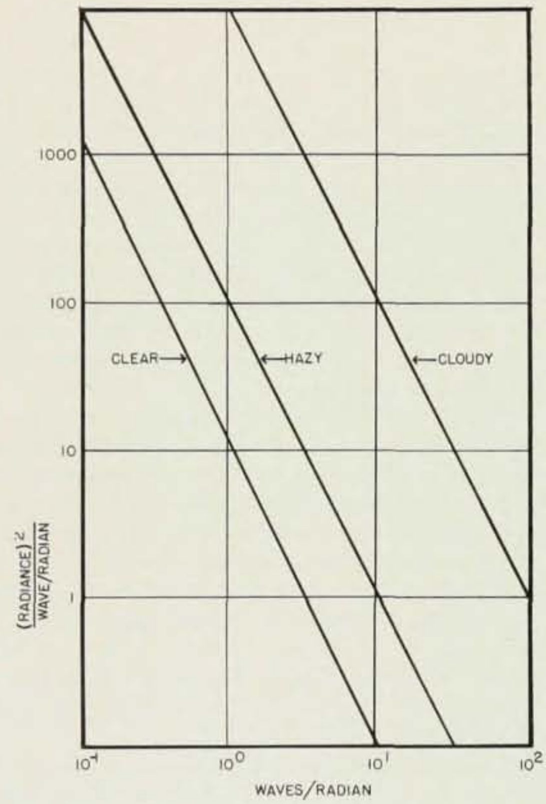



*Fig. 3c.* Wiener spectrum, infrared-night sky radiation.

ONE may wonder what the relation is between this power spectrum and the "power" value an ordinary light meter records when held before an object. The stationary light meter can indicate only the average or dc value of brightness; it does not show the spatial distribution of this power. It is well to emphasize that the existence of a spatial power spectrum does not imply that the responsible physical process really consists of a superposition of harmonic oscillations. These fluctuations are simply defined in the manner of a power spectrum.

Whereas an artist might be interested in preserving all the details of a cloudy sky with true fidelity, an observer searching for a distant airplane isn't the least bit interested in cloud structure. His target will have its energy distributed through a very small angular dimension in comparison with the clouds. Small objects have a Wiener spectrum which is essentially flat as far out as a frequency of half the reciprocal of the object's angular width (Fig. 3). On the other hand, cloud energy is mostly modulated at lower space frequencies (i.e., the clouds have large angular dimensions). Proper optical systems for the artistic photographer and the aircraft observer should have dissimilar specifications, because each deals with dissimilar object power spectra.

With the tools of information theory, one can make sure of getting the most efficient and economical device for observing things of a specialized character. Just how efficient a device can become is indicated by the experience of automatic star trackers. They are able to follow stars whose energy is only 1/100 000 that of the surrounding daylight sky flux.

Even though information theory leads to a better understanding of optical processes, it is not a significant tool leading to new discoveries in the sense that quantum mechanics has affected nuclear physics.

Besides providing descriptions of backgrounds and targets through Fourier transforms, the trend now is to specify lenses and photographic films in the same manner. Traditionally the index of lens quality is the specification "blur circle", the smallest size image which can be formed from a distant point object. Actually the distribution of energy within this spot varies between cameras, and the spot might not even be a circle. One lens might generate a uniformly illuminated patch and another might show a faint halo surrounding a small bright nucleus. Failure to account for the image character results in cameras with the same blur-circle size performing quite differently under service conditions.

For testing optical resolving power, line patterns are superior to points. Charts having a multitude of black-and-white line groups of varying degrees of fineness have long been standardized. Resolution seems to be so understandable that persons are inclined to regard it as a fundamental quality parameter, for nothing seems more straightforward than to evaluate the lens on the minimum line size found separated on the film receiver. However uncertainties still occur because the boundaries between white and dark lines will never appear as sharp edges. Results are influenced by the contrast between the white and the black. Finally there is the disturbing situation that good resolving power alone does not establish the quality of image definition.

There is still another approach to analyzing lenses which recognizes the fact that the brightness distribution of any pictorial scene can be synthesized as a series of sine waves. What is needed is knowledge of how the lens will transmit energy at each space frequency. Testing can still be done against targets having parallel black and white lines, but now the projected image at the focal plane is scanned by a tiny calibrated photocell.

As the minute detector traverses the image pattern, the observed image intensity changes as if it were the sum of two components—the average or dc level and a modulated portion which adds or subtracts from the average level. Prior to the test the detector scans the target chart by itself. Actually we seldom care about the dc response of the system, just so long as the average level is high enough to expose the film or activate the detector. The dc component is of little interest because only the modulated portion of the image can convey information.

A lens will always cause some diffusion of the image, thus throwing some of the light from a white area into the spot where a dark bar should appear. Consequently, although the patterns on the image and object planes may have the same average intensities, the amount of modulated light may be quite different. The ratio of the modulated light intensity between object and image is called the transfer value, a dimensionless number between one and zero. A summary plot of transfer values for all the test-bar spacings (space frequencies) is called the "transfer function".

Even a pinhole camera has a transfer function. A more typical lens is represented by the curve shown in Fig. 4a. One notes how the efficiency inevitably drops at the higher space frequencies, giving cause to describe the lens as a low-pass filter. This attenuation effect means that fine-grained object details will be resolved only if they have a great deal of contrast to their surroundings.

Lens quality isn't the only influence on the flow of optical intelligence. There will be further degradation because of the granularity of the photographic film or the photoelectric detector. Transfer functions have been measured for most of the common films (Fig. 4b). These function curves won't tell the photographer anything about the sensitivity of his film, for they are based on the assumption that the film has been correctly exposed and developed to a state where film response is linear to the input.

One cannot consider only those optical parts behind the telescope objective lens, but must make allowance for the mass of air along the line of sight. Light waves, on passing through layers of atmospheric turbulence, become distorted and give rise to effects of scintillation, image dancing, and image broadening. Actually the atmospheric air mass sets the present limit on telescope performance. Under the best conditions the earth-bound astronomer can never obtain resolution better than about one second of arc. More typically the dis-

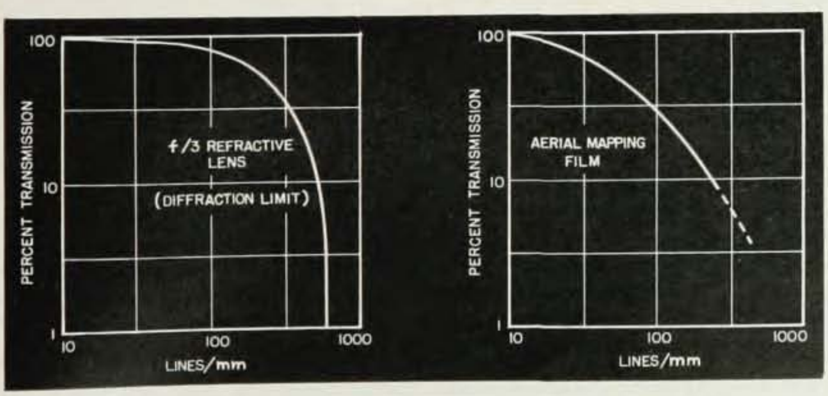
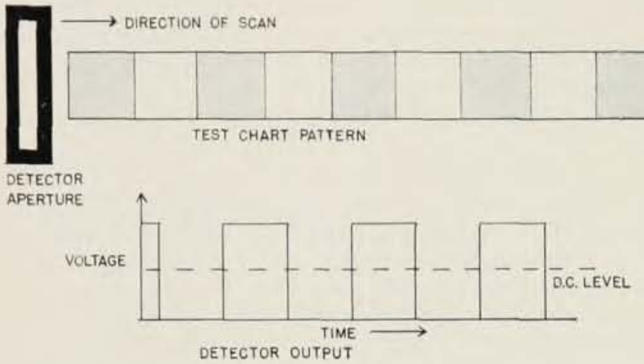



*Fig. 4.* A comparison of transfer functions for a typical lens and a special-purpose film.



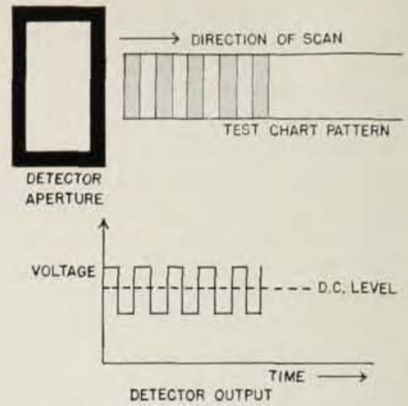




*Fig. 5.* Effect of the relationship between the size of the detector aperture and the spacing of bars in a test pattern on the operating characteristics of a photoelectric cell.

tortion will be ten or even a hundred times worse. By comparison the two-hundred-inch diameter of the Mt. Palomar telescope could resolve 1/30th of a second.

This scattering because of the air is random in motion, and the probability of the image appearing at a point displaced from the average position can be expressed mathematically. Once the amplitude of the image excursions is known, an exact transfer function can be calculated for the effect.

Although the influences on optical performance may represent diverse phenomena, nearly all factors can be equated to the common terms of a transfer function. Besides those effects already mentioned, a system is degraded by platform vibration, target movement, granularity in the visual display machine, and many others. To estimate net system performance one must take the initial value of target energy at a given space frequency and multiply it successively by the transfer values of every data-transmission link. A procedure akin to this is used to compare high-fidelity sound systems. A standard audio frequency is taken and the system loss calculated due to the needle cartridge, preamplifier, amplifier, and loudspeaker. Since a real target, or real music, is defined by many sine-wave oscillations, this multiplying operation should actually be extended to all frequencies for the complete picture.

EVERYTHING brought out so far could be applied equally well to photographic or photoelectric sensors. When a television tube or a photoelectric cell is employed, new effects are introduced by the detector size, the scan rate, and the amplifier. Now those optical elements operating on a space-frequency basis must be combined with those components referenced to a time frequency. The conversion can be made with the common denominator, angular scan rate.

As a detector moves across a bar pattern, its electrical output will faithfully reproduce the modulated target energy, provided the target's angular dimensions exceed those of the detector. Confronted with smaller bar spacings than the cell size, the cell can no longer record the changes as effectively as before, although the dc level remains as high as ever. This is much the same degradation process previously described for a lens-blur circle, except now the operation is in the time dimension (Fig. 5).

Examination will show that for the coarse pattern the cell can be subtended either by an all-white or all-black area at certain instants. Transmission is then virtually 100% for that space frequency. On another finer pattern the two extreme detector views may be four white—five black, five white—four black; thus the cell size only allows receipt of 20% of the original modulated energy. The transfer function for a rectangular aperture corresponds to the formula $\sin X/X$, where $X$ is the space frequency.

One outgrowth of information theory has been the attempts to correct for the nonuniform response of an aperture. Theoretically a detector can be devised with shading and masking of such nature that its transfer function would be more uniform than that of a rectangular opening. Technically the procedure is difficult and causes a loss in sensitivity. The simplest way of compensation is to use an amplifier whose response is just the inverse of the aperture transfer function. Where the aperture transmission drops, the amplifier gain increases. Ultimately so much gain is required at higher frequencies that any signal improvement is masked by noise.

A very significant change occurs to the aperture transfer function if a picket-fence mask is placed at the focal plane. When the space frequency of the object matches the space frequency of the mask or reticle spacing, the transfer value will be at the maximum. Optimum system performance is indicated by the sharp reinforcement peaks that now appear on the aperture transfer-function curve (Fig. 6). Objects with either higher or lower space frequencies are effectively fil-

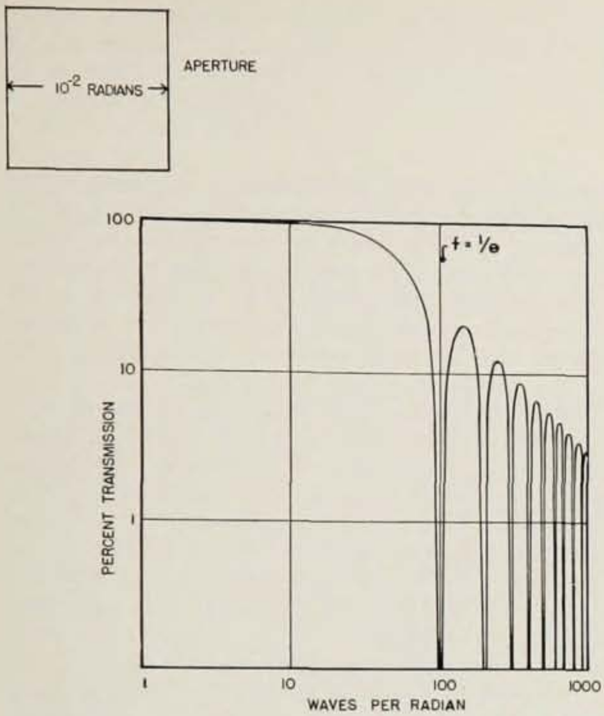



*Fig. 6a.* Transfer-function curve for detector aperture measuring 0.01 radian.

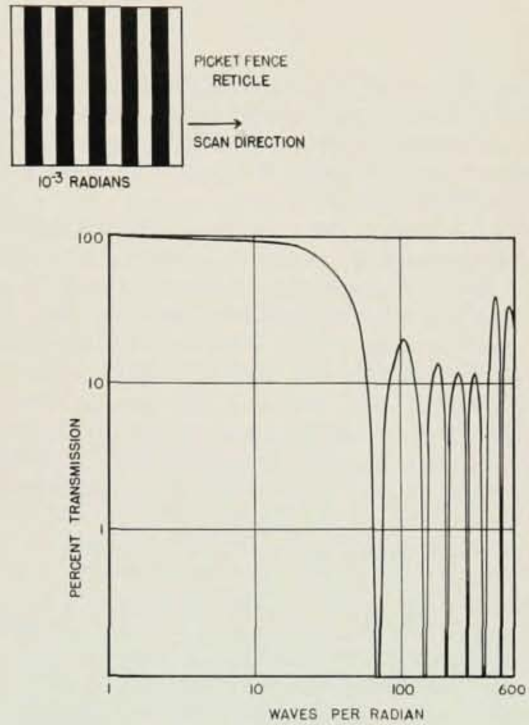



*Fig. 6b.* Transfer-function curve for picket-fence reticle having six slits each measuring 0.001 radian in width.

tered out, provided the electrical band pass of the amplifier is made to match.

Nearly every infrared guided missile, star tracker, surveillance scanner, and optical tracker will incorporate a reticle just before the detector at the focal plane. Although other techniques, including color filtering, will also be used, space filters usually are the most powerful agents in reducing unwonted background interference. For the example of a star tracker designed for a point target, the reticle spacing can be fine enough to reduce noise from a cloudy daylight sky to 1/5000 the value otherwise encountered.

To understand the principle of a space filter, remember how the Wiener spectrum of a sky background and a point target differ: the sky brightness decreases in inverse proportion to the square of the space frequency, whereas the star-image spectrum remains essentially flat out to very high space frequencies. With a reticle we push the response of the optical device out to those high frequencies where signal to noise (i.e., the sky noise) is the best.

Physically, the reticle for a guided-missile head may be but 1/4 inch in diameter with lines measuring less than 1/1000 of an inch photoengraved on the surface. An early design, soon abandoned, was simply the sunburst or spoked-wheel pattern. It did too good a job of modulating horizon lines. Next was a checkerboard-like pattern, also found to chop cloud edges and horizons. There has been no standardization of reticle designs; some successful ones have their bars shaped like the arms of a spiral, or as the zigzag lines sometimes called the "indian blanket".

With the modern advances in satellite-borne cameras, infrared mappers, laboratory spectrophotometers, and aerial photography, we have ample evidence of the current surge in optical technology. Yet these improvements do not represent any radical changes in thought or design of a decade ago. Largely they result from the existence of new optical glasses, better detectors, light-weight electronic parts, new processes, and electronic computers for calculating lens design. Future improvements in optics cannot continue to depend so heavily on the introduction of new components. Rather, the emphasis will be on making better use of the techniques now available. For that task, information theory will be the prime tool.