within a few plasma periods. He presented numerical calculations for the development of the instabilities in the nonlinear regime. J. R. Pierce provided us with some of the wealth of examples of instabilities and growing waves in electron beam devices. J. Dawson (Princeton) discussed plasma oscillations of a large number of electron beams and E. G. Harris (University of Tennessee) spoke on instabilities of transverse as well as longitudinal waves in plasmas with anisotropic velocity distributions.

DURING the final sessions on Saturday morning, E. N. Parker (University of Chicago) showed that the two-stream electrostatic instability should lead to the formation of shock waves in tenuous plasmas. The characteristic distance is the relative stream velocity times the ion plasma period. He also showed that the presence of a magnetic field introduces another coupling and a characteristic shock thickness of the ion Larmor radius. C. Oberman (Princeton) discussed some properties of the transmission, reflection, and radiation of bounded plasmas in a strong magnetic field. J. Dawson (Princeton) described results of numerical calculations on the thermalization of large-amplitude plasma oscillations. They show that for an initially cold plasma, the ordered motion was 60 percent randomized within the first three oscillation periods. For an initially tepid plasma, 90 percent randomization of the wave motion occurred in the first oscillation period. An interesting by-product of the calculation was the observation that a few particles gained energies in the order of 10 times the average energy. P. Sturrock (Stanford) discussed nonlinear effects, pointing out how the Hamiltonian formalism leads naturally to expressions for power transfer and frequency shifts in terms of "collisions" between various normal wave modes in the plasma.

S. Buchsbaum (Bell Laboratories) discussed the experimental microwave generation of a column of plasma in a cavity, and the use of higher cavity modes to measure properties of the plasma.

The proceedings are to be published in the *Journal of Plasma Physics—Accelerators—Thermonuclear Research* which is Part C of the *Journal of Nuclear Energy,* and subsequently in book form by Pergamon Press. A summary of some of the ideas presented at the Institute was given by M. Rosenbluth. The following is a complete version of that summary.

PLASMA PHYSICS
(*"quantum" and "classical"*)

*By **Marshall N. Rosenbluth***

FIRST of all I would like to thank our hosts from the University of Washington and from the Boeing Airplane Company. In particular, Dr. James Drummond has labored indefatigably for many months to bring about this conference. I think we have all been impressed by the program he has provided for us and by the general excellence of the papers presented at this meeting. In fact this very excellence provides me with a considerable problem since this field is so diverse and has been so well covered by the preceding speakers. I am faced by a sort of insoluble "many-subject" problem which I can only treat in a very cursory fashion which may best be described as the Random Phrase Approximation.

One thing that has been quite apparent at this meeting is a dichotomy between two rather different subjects which I will describe by the somewhat inexact titles of "quantum" and "classical" plasma physics. In general those familiar with one of these fields are quite ignorant of the other. This is certainly true in my own case so I hope that my simple-minded comments on quantum plasmas will be forgiven. One may hope that the cross-fertilization provided by this meeting will prove fruitful.

I would now like to describe in a qualitative way some similarities and contrasts between the two disciplines. The basic similarity is, of course, that in both cases we are concerned with plasmas—that is to say a many-body system in which the particles interact through electric and magnetic forces. However the basic physical situations of the plasmas are quite different. Quantum plasmas are primarily studied in connection with solid-state physics while classical plasma studies are motivated by fusion reactor work, electron tube research, and astrophysical applications. It is the conditions prevailing in these physical situations which of course determine the nature of the disciplines.

In the first place, in the quantum plasma the electrons are degenerate which means that the exclusion principle is important and that spin and exchange effects must be considered. An equally important contrast is that in the classical plasma the potential energy between particles at an average interparticle separation is very small compared to the mean thermal kinetic

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energy. Indeed this must be so or in a classical problem the electrons would all merely fall into the ions! This means that only the electromagnetic fields of many particles moving together coherently can be of importance—hence the validity of equations stressing the collective behavior of plasmas. Two equivalent approaches of this type are the random phase approximation of Bohm and Pines and the collisionless Boltzmann equation.

On the other hand, in metals the potential energy is somewhat greater than the Fermi kinetic energy. Actually they are quite comparable in magnitude. Nonetheless the basis of the calculations usually made, such as the high density expansion or the random phase approximation, is again that the kinetic energy is large. It is paradoxical under the circumstances that such good results have been obtained.

Thus we may conclude that the basic physics of quantum plasmas is more complicated both due to quantum effects and to the fact that neither a weak nor a strong coupling approach is obviously justified. On the other hand while the basic equations of classical plasmas are simple, the questions which we ask of them are very complex. Thus the study of quantum plasmas is primarily concerned with the properties of a true thermodynamic equilibrium and with small excitations around such an equilibrium. Because of the complex physics involved the answers to such problems are not simple. On the other hand the true thermodynamic equilibrium of a classical plasma is most uninteresting —a Maxwell distribution uniformly filling all space (this is true irrespective of the presence of a static magnetic field).

Thus the heart of the classical plasma problem is that it is basically a nonequilibrium problem. For instance the inhomogeneity of the plasma is usually of importance. At this point it appears useful to coin a phrase to describe situations which are often of interest—*metaequilibrium*. We have said that the only thermodynamic equilibrium is a uniform Maxwellian distribution. On the other hand we have also pointed out that since the interparticle interaction is usually rather small it follows that the collision frequency is very low compared to the collective frequencies of the plasma. However it is only these close collisions which drive the system to thermodynamic equilibrium. Hence we may define a metaequilibrium as any time-independent solution of the collisionless Boltzmann equation and Maxwell's equations. The effect of close collisions is then to make an adiabatic transition between different metaequilibrium states. It is the existence, nature, and stability of these metaequilibria with which we are concerned. Since these are not true equilibria we are not able to use many of the powerful methods of thermodynamics and statistical mechanics.

This difference in the problems to be studied is clearly seen in the nature of the experimental information available. For the quantum plasma physicist the experimental facts are by and large well known. His plasmas are contained in nice stable pieces of metal. On the other hand the classical plasmas which exist in nature are ordinarily turbulent and have resulted from very complicated processes. In fact one can easily prove by the virial theorem that any classical plasma which is not confined by rigid material walls, externally imposed magnetic fields, or gravitation, must explode. A relatively simple metaequilibrium is however provided by the natural radiation belts surrounding the earth. The experimentalist who wishes to create a laboratory plasma is also beset by many problems since as we shall see it is only with very carefully tailored metaequilibria that he may hope for stability.

Moreover, fantastic purity is required in order that cooling by impurities be tolerable. Thus a monomolecular layer of wall material will destroy a fusion plasma. The result of these difficulties is that the experimental information is still quite fragmentary and often deals with situations which are far from the idealized ones envisaged by the theorist.

In summation, it would appear that the principal task of quantum plasma theory is the development of an intermediate coupling approach which can make a quantitative check with the experimental observations. In the classical case on the other hand the weak coupling theory is valid and the problem may be stated as the development of the statistical mechanics of a system in which only collective modes are important. In particular the question arises to what extent these modes, without collisions, are capable of producing a true thermodynamic equilibrium.

I WOULD now like to describe in somewhat more detail the present state of development of the theory of classical plasmas. For the sake of definiteness I shall tie the discussion to the consideration of quiescent inhomogeneous plasmas confined by a static magnetic field as in proposed fusion reactors. First a brief discussion of the justification for the collisionless Boltzmann equation. This can be written

$$\frac{\partial f}{\partial t} + v \cdot \nabla f + \frac{e}{m}(E + v \times B) \cdot \nabla_v f = 0 \qquad (1)$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t} \qquad (2)$$

$$\nabla \times \nabla \times E + \frac{1}{c^2}E = -4\pi e \int \frac{\partial f}{\partial t} v d^3v. \qquad (3)$$

This set of equations is of course exact if given the proper interpretation, namely that the distribution function f consists of a set of δ functions at the points in phase space representing the positions of the particles. Then of course it is not very useful. Let us now conceptually smear out the δ functions over a region in phase space large compared to the interparticle distance but small compared to regions of macroscopic interest. Then the same equations are applicable but f is now a continuous distribution function; e.g., $e^{-mv^2/2kT}$. This is the usual collisionless Boltzmann equation. Mathematically we may accomplish the approach to a continuous fluid by letting n, the particle density, become infinite in such a way that ne, nm, and v remain constant. The exact equations may then be expanded in powers of $1/n$. The dimensionless expansion parameter turns out to be the inverse of the number of particles in a sphere of radius the Debye length $(mv^2/4\pi ne^2)^{1/2}$. For typical situations of interest this parameter may be 10^{-6}. To lowest order one obtains the collisionless Boltzmann equation and to next order the Fokker-Planck equation. Time scales for the collisional processes described by the Fokker-Planck equation are then 10^6 times as long as the collective processes described by the collisionless Boltzmann equation.

Let us now turn to the question of the existence of static metaequilibrium solutions of equations (1)–(3). The operator in equation (1) may be interpreted as the total time derivative of f moving through phase space along the particle orbit which is governed by E and B. Hence if closed orbits exist in phase space a time-independent solution of (1) is simply that f be constant along such an orbit. In the case of magnetic fields possessing sufficient symmetry the existence of closed orbits can be shown and solutions of the above equations explicitly given.

For physically realizable fields with imperfect symmetry the orbits do not exactly close and an exact confined metaequilibrium does not exist. However some beautiful work by Kruskal and Gardner has shown that as long as the changes in the field are small within a particle radius of gyration then the orbits come extremely close to being closed. This has been experimentally verified by Lauer and co-workers at the Lawrence Radiation Laboratory, who have shown that the decay positrons from radioactive neon remain confined within a mirror machine for many millions of traversals. Thus it may be considered to have been proven that confined static metaequilibria exist for all practical purposes.

Next we must study the stability of these equilibria. To date almost all work in this direction has been based on a linearized theory. That is to say we consider small perturbations around a static metaequilibrium which is described by a distribution function f_0 and fields E_0 and B_0. If we denote the perturbations by subscript 1, then the linearized form of equation (1) becomes

$$\frac{df_1}{dt} = -\frac{e}{m}(E_1 + v \times B_1)\cdot\nabla_v f_0, \qquad (4)$$

where the time derivative is to be taken along the particle orbit in the unperturbed field. By performing the time integral and substituting in (3) an integro-differential equation is obtained for the perturbed field E_1.

In general this equation is difficult to solve because of the complicated nature of the orbits in an arbitrary field. In two special cases the orbits are simple and the analysis has been performed. The first case, to which I will return later, is that of an infinite homogeneous plasma in a constant magnetic field. Here the orbits are simple spirals and the integro-differential equation reduces to an algebraic dispersion relation for the frequency of the normal modes. The second simple case is obtained by restricting our attention to disturbances of low frequency and long wavelength such that the fields which a particle sees vary only slightly during a Larmor period. In this limit the particles remain "frozen" on magnetic field lines and there exist adiabatic invariants of the motion, i.e., the magnetic moment and longitudinal action integral, such that the equations assume a simple character. In fact in this case the equations of fluid magnetohydrodynamics, such as would apply to a liquid metal, remain valid except for the adiabatic law of pressure change. This becomes more complicated because of the existence of unimpeded heat flow along the lines of force but one is still able to construct a variational principle to test for stability based on the concept of virtual displacement of the field lines. One is led to the interesting result that a plasma is more stable with respect to these hydrodynamic-type modes than a liquid metal would be. Nonetheless the great majority of plasma containment devices are unstable to such perturbations. However, certain stable configurations have been found, e.g., the helically wound stellarator, the inverted pinch, the cusp, and the mirror machine with conducting end-plates.

LET us now return to the consideration of an infinite homogeneous plasma where the dispersion relations may be derived exactly. These are very rich dispersion relations and much has been learned from them. In the first place one can show easily that if the distribution function is isotropic and a decreasing function of energy, i.e., if f is only a function of v^2 and if $\partial f/\partial v^2 < 0$, then all the modes of excitation are stable. Hence we know that collective oscillations by themselves cannot play the role of collisions in producing a Maxwell distribution.

A great many modes exist. In addition to the hydromagnetic modes we have already discussed there are electrostatic plasma oscillations, cyclotron waves, transverse electromagnetic waves, and a variety of others. We will refer to these as the high-frequency modes.

Let us now look at the case where the distribution function is not isotropic. For example there may be an asymmetry in the components of the pressure tensor perpendicular and parallel to the magnetic field or there may be beams of particles moving along the field lines. Such beams might be produced by "runaway" electrons which have been accelerated by a large electric field.

Where an anisotropy exists the high-frequency modes often become overstable, that is to say subject to growing oscillations. The mechanism for this is rather interesting. It depends on a trapping of individual particles by waves. Thus when the phase velocity of the wave equals the velocity of a particle, the latter moves along in phase with the wave and becomes trapped by it. If it was initially going faster than the wave it will give up energy in the course of the trapping and thereby feed a growing oscillation. If on the other hand there are more slow particles then the wave will be damped. Hence if a beam of particles is present the plasma is likely to be unstable with respect to waves moving slightly slower than the beam. Fortunately in the limit that the anisotropy becomes small the instabilities either vanish or their rate of growth becomes negligibly small.

The existence of these instabilities poses great problems in obtaining a hot plasma. On the one hand the distribution function must always be nearly isotropic in order to maintain stability. On the other hand the processes of breaking down the gas and subsequently confining and heating the plasma are likely to be rather violent, involving large accelerating electric fields, and this violence may well lead to anisotropies. Hence a principal problem for the experimentalist is to learn the art of gentle handling of plasmas.

There remains one gap in our understanding of the linear stability of confined plasmas. That is the behavior of the high-frequency modes in an inhomogeneous plasma. I would now like to describe some preliminary conclusions of recent work by Nick Krall and myself. Our study of the homogeneous case has led to an understanding that except for the effects of the trapping process described above the plasma may be thought of as a substance with a real dielectric constant for which the square of the frequency remains real. Hence the introduction of a small inhomogeneity in the plasma will only lead to a frequency shift rather than overstability except for the effects of trapping.

However, there is a new possibility for trapping which exists in an inhomogeneous plasma—namely the particle drifts and currents perpendicular to the magnetic field. Such currents are of course essential to a confined plasma since it is the $j \times B$ force which supports the plasma pressure gradient. Thus we are led to expect that waves with a phase velocity comparable to the plasma drift velocity may become unstable.

Now the plasma drift velocities are much smaller than their thermal velocities. In fact if we describe the inhomogeneity as being of magnitude ϵ, then the drift velocities are equal to ϵ times the thermal velocities. Roughly speaking ϵ is the fractional charge of the magnitude of the field in a particle radius of gyration. The condition for trapping is then that the phase velocity of the wave equal the drift velocity

$$v_{\text{phase}} = \omega\lambda = \epsilon v_{\text{thermal}}. \qquad (5)$$

Here λ is the wavelength of the disturbance and ω is a characteristic frequency of the plasma such as the Larmor frequency or the plasma frequency. It follows then that the possible unstable high-frequency modes will have a very short wavelength for weakly inhomogeneous plasmas, proportional to ϵ. A more detailed analysis seems to support the above qualitative arguments and thus leads to the conclusion that even a hydrodynamically stable confined plasma with no pronounced anisotropies may be subject to very short wavelength instabilities.

We now come to the broadest area of ignorance in the entire field—namely the question of nonlinear effects. While we may be able to ascertain that an equilibrium is subject to unstable small-amplitude oscillations we have very little idea what the ultimate effect of the instabilities will be. Presumably they will always act in a direction to smooth out the inhomogeneities or anisotropies which are driving them. They are likely also to induce a kind of turbulence so that the plasma is far from the sort of static metaequilibrium we have been discussing. This question is of particular interest for the short wavelength instabilities which are characteristic of the high-frequency modes, since nonlinear effects must predominate when the amplitude of the perturbation exceeds the wavelength.

Some numerical work on a simple model by Dawson and Bunemann suggests that what may happen is that once the nonlinear stage has been reached then the wave quickly breaks and the energy is distributed to even shorter wavelengths eventually approaching some sort of thermalization among the particles. If this picture is correct then the plasma may be thought of as undergoing very localized boiling. A very crude estimate of the effects of such boiling can be made by constructing a diffusion coefficient equal to the square of the wavelengths of the instability times its linear growth rate. Because of the short wavelengths involved, such diffusion coefficients are usually very small. On the other hand, Sturrock has shown that there is also some tendency, at least in the early nonlinear phases, for the energy to move towards longer as well as shorter wavelengths. If a sizable amount of energy can be built up in long wavelength oscillations then a more rapid diffusion would, of course, result. In addition to diffusion the possibility that such turbulence might lead to sizable coherent radiation being emitted from the plasma must also be considered. In view of the failure over many decades to produce a satisfactory a priori theory of ordinary hydrodynamic turbulence it seems clear that the answer to these questions must be provided in large part by experiment.

To sum up, I think anyone would have to be impressed with the vast amount of work and new information which has come forth in the past few years concerning plasmas as well as other many-body systems. In the field of quantum plasmas we now have a good theoretical understanding of the basic equations which govern metallic plasmas. Even such an exotic effect as superconductivity is explained which appears to be a really remarkable triumph of the theory. The theory of high-temperature classical plasmas, at least in the neighborhood of quiescent metaequilibrium states, has been well formulated. And while no one is yet close to fusion power, nonetheless kilovolt plasmas have been produced which persist for times several orders of magnitude longer than the time scales for the simple collective instabilities which beset the earliest work in this field.