The Foundations of

MECHANICS *and* THERMODYNAMICS

A conference report by ***E. A. Kearsley and M. S. Green***

ON October 21–23, 1959, a Conference on the Foundations of Mechanics and Thermodynamics was held at the Cosmos Club in Washington, D. C. The idea for this conference arose at a series of informal discussions at the National Bureau of Standards among workers in the fields of continuum mechanics and statistical mechanics. Although these subjects can both be classed as branches of mechanics, associated with each a vocabulary and mode of thinking have developed such that the relationship between these disciplines is only vaguely discernible. The conference was designed to shed a little light on this problem, in particular by exploring and comparing the foundations and premises at the heart of each.

The formal part of the conference consisted of five half-day sessions. One prepared speaker was scheduled for each session to introduce the subject, but the major part of the time was devoted to informal speeches, discussion, and diatribes from the floor. All the speakers enjoyed considerable interruption and comment from the floor, to such an extent that the very able moderator, Clifford Truesdell of Indiana University, frequently felt called upon to declare and enforce a "five-minute truce" during which the speaker could present a particularly subtle or difficult point without interruption. Prior to the conference, it had been feared that the unusually large amount of time scheduled for comments from the floor might result in embarrassing silences. This fear proved quite foolish; every session of the conference was marked by such a lively discussion that it was difficult to conclude the sessions on time. At its conclusion, each session was characterized in verse by Barry Bernstein of the Naval Research Laboratory.

The program for the five sessions was as follows:

Walter Noll, Carnegie Institute of Technology, "The Mechanics of Continuous Media"

John Ross, Brown University, "The Statistical Mechanical Theory of Irreversible Processes"

Bernard D. Coleman, Mellon Institute, "The Thermodynamics of Deformable Media"

Elliott W. Montroll, University of Maryland, "The Phenomenological Approach to Irreversible Thermodynamics"

Richard A. Toupin, Naval Research Laboratory, "World Invariant Electrodynamics"

The first session began with a short welcoming statement by Charles Herzfeld of the National Bureau of Standards, followed by the moderator's introduction which suggested the subject of the conference in the form of eight questions, viz.:

1. In phenomenological terms, what are the "forces" and "fluxes" of the Onsager relations?

2. Are the equations of irreversible thermodynamics sufficient to determine motions, or do new principles remain to be found?

3. What are the independent variables of thermomechanics?

4. What are the basic principles of entropy production for large irreversible deformation?

5. Within statistical mechanics, for what circumstances, if any, is the Boltzmann equation a valid approximation?

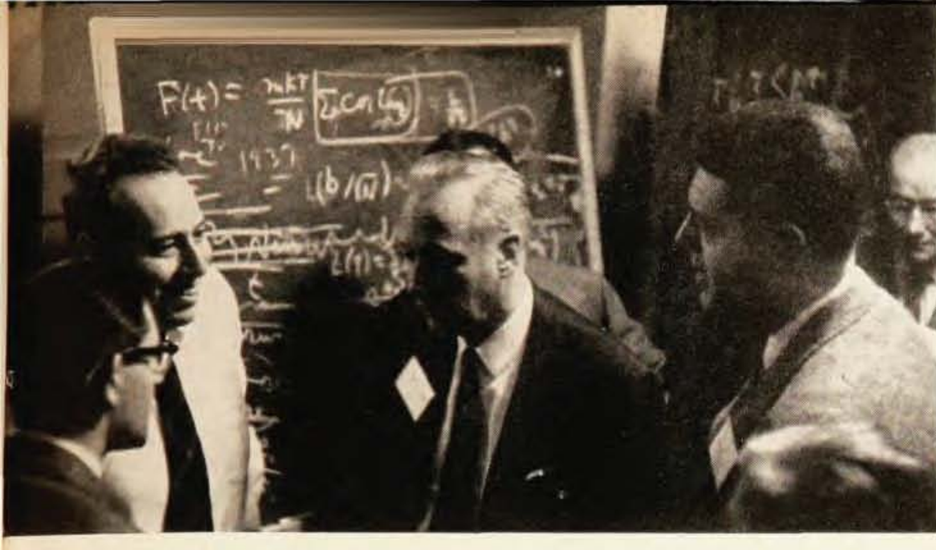
6. What equations govern the temporal evolution of molecular distribution functions in imperfect gases and liquids?

7. What are the hydrodynamical laws according to the kinetic theory?

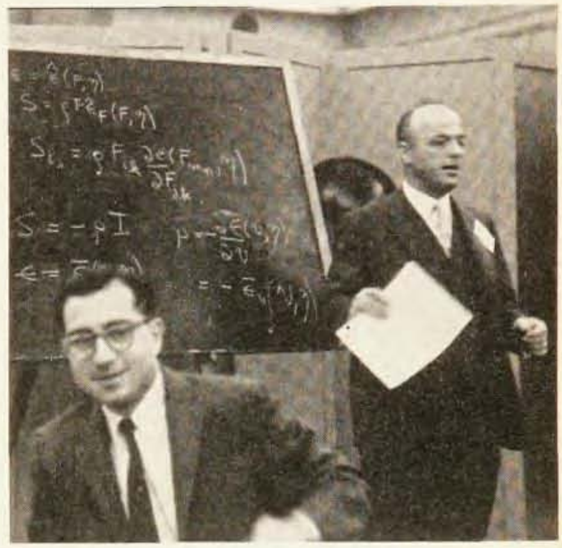
8. What is an "observer" and what are the properties of invariance appropriate to both electromagnetics and mechanics?

Walter Noll's talk, The Mechanics of Continuous Media, began with a clandestine revelation of the secret cult of the Body $\mathcal{B}$—a mystic rite which always begins, "A body $\mathcal{B}$ is a smooth manifold of elements . . . ." The purpose of this rite is to formulate continuum mechanics using rigorous mathematical definitions and utilizing modern sharp mathematical tools. The value of this approach is that it emphasizes the premises and

Elliot A. Kearsley and Melville S. Green are members of the Heat Division of the National Bureau of Standards.



Left: Harold Grad, Elliott Montroll, Robert Marvin, Stephen Tamor, and Johannes Burgers. Below: moderator Clifford Truesdell calls a truce while speaker Bernard Coleman smiles with relief.
Photos by John Hoffman, NBS



assumptions of the theory and their relationship to the conclusions. After illustrating the method by "rediscovering" Cauchy's laws of equilibrium in continuum mechanics, it was applied to a study of the form of the constitutive equations of a material, that is, equations on the material properties of the body, such as the caloric equation of state, or stress-strain laws. This introduced the so-called principle of material objectivity which corresponds to the idea that material properties of a body are independent of its orientation in space. Finally materials were classified in terms of associated isotropy groups; an isotropic material is characterized by constitutive equations unchanged by rotation, a simple fluid by constitutive equations unchanged by the complete unimodular group of transformations.

It was interesting to observe that, although the ideas of objectivity and isotropy seem vaguely obvious, they are in fact so subtle that a clear and unambiguous statement of these ideas requires exceedingly sophisticated mathematics. The discussion from the floor underlined this subtlety.

The relationship between statistical-mechanical theory and rational continuum theory of matter was the heart of the talk by John Ross. Statistical mechanics may be used to derive or to discover the phenomenological equations which are the bare bones of a rational theory. Statistical mechanics may be used to determine from molecular data the phenomenological coefficients, or, to use the language of rational mechanics, the constitutive relations. Finally statistical mechanics may be used to determine the limits of validity of either the phenomenological equations or of the molecular expressions for the phenomenological coefficients.

Two examples were used to illustrate this relationship, one from the area of equilibrium statistical mechanics and one from the area of nonequilibrium statistical mechanics. The first of these was the well-known van der Waals' equation of state $(p + a/v^2)(v - b) = RT$. It was pointed out that the phenomenological constants a and b have a particular meaning only in terms of molecular statistical mechanics.

The nonequilibrium example was the statistical-mechanical derivation of the Boltzmann equation from a point of view developed by Mori, Oppenheim, and Ross. Among the participants were a number of proponents of differing points of view, and many lively discussions developed.

Bernard Coleman's talk, The Thermodynamics of Deformable Media, utilized the rite of the Body $\mathcal{B}$ to explore thermostatics. The talk began with a mathematization of Gibbs' concept of stable equilibrium of an isolated system. In particular, a fluid body is assumed for which a caloric equation of state exists so that an internal energy density is defined for each element of a body in terms of the specific volume and entropy of the element. A "state" of the body is defined as a particular assignment of configuration of the elements and an entropy density for each element. Global stability of a state occurs when any other state with the same total volume and energy has a greater total entropy. It follows that for a state to have global stability it is necessary that derivatives of internal energy density with respect to entropy density and with respect to specific volume shall both be constant. Finally, sufficient conditions for the uniform state to be globally stable are that a certain convex inequality in the internal energy density shall hold. This approach was then generalized to cope with nonfluid bodies for which the internal energy density is an isotropic function of deformation gradients. Local stability of a state is then defined in terms of minimizing a quantity similar to the Helmholtz free energy. Thermal equilibrium seems to associate in a natural way with this concept of local stability. From the general discussion provoked by this talk, it seemed that the two disciplines, statistical mechanics and continuum mechanics, come closest to overlapping on these subjects.

Elliott Montroll's talk emphasized the importance of the equilibrium autocorrelation function to understanding irreversible phenomena in general and, in particular, for the computation of transport coefficients. This function has a repetitive character for a system composed of a finite number of particles. As an example, the autocorrelation of the momentum of a molecule in a perfectly harmonic crystal was shown to be an almost periodic function for a finite crystal becoming aperiodic in the limit of an infinite number of particles. It was suggested that in this transition of the autocorrelation function from repetitive to aperiodic lies the answer to the puzzling irreversibility of reversible dynamic systems. This point gave rise to perhaps the liveliest and loudest discussion of the conference, so that the speaker was able to conclude his talk only under an extended "truce" called by the moderator. The conclusion consisted of a sketch of the use of these autocorrelation ideas in the study of a quantum electron gas. Unfortunately the speaker had time to draw only a few toron diagrams before the meeting broke up for lunch.

A theory of continuum electrodynamics, the subject of Richard Toupin's talk, combines the disciplines of continuum mechanics and electromagnetic theory. The concept of "observer" associated with electromagnetic theory is different from that of classical mechanics, which suggests that a more general principle of material objectivity is required. The conservation laws of physics can be expressed independently of the geometric structure of the space-time manifold assumed and thus, independently of the concept of observer. In this theory, applicable to polarizable and magnetizable media, the postulates and equations were separated into two classes, conservation laws independent of the geometry of space-time and constitutive relations depending on or employing a specific geometric structure. The conservation laws assume only an affine space-time—i.e., an operator called a shifter is assumed to exist which allows comparison for parallelism of vectors separated in space-time. It then follows that each conservative field possesses a potential.

The remainder of the talk was concerned with the constitutive relations, which involve assumptions on the particular geometric structure of space-time. In particular, the complications of relativistic continuum mechanics were avoided by using the classical concepts of absolute time and allowing the equations to be influenced by an "aether wind", in the spirit of classical physics. Following a series of carefully stated assumptions an extremely general equation of entropy production resulted, consistent with the principles of mechanics and electromagnetism. Innumerable special theories can then be based on this equation along with constitutive equations for the various dependent fields such as heat flux, magnetic flux density, stress, etc., in terms of the independent fields of the theory. A special case was mentioned as an example, a rational and unified theory containing piezoelectricity, photoelasticity, and the Faraday effect.

IT is certainly fortunate that the laws of physics did not have to be settled by a majority vote of the conference, for it is doubtful that a majority existed on any of the many questions that arose. This fact is not surprising since the participants included physicists, chemists, and mathematicians. It is interesting that the most vigorously contested points were usually regarded as obvious and intuitive by the defending faction. While the airing of these questions certainly did not settle them, it was an enjoyable experience and instructive to everyone.

In fact, the conference has already been reported in a much more succinct and readable form by Barry Bernstein of the Naval Research Laboratory, who concluded each session with the following verses:

Reactions of a Fixed Observer (or More than Molecular Chaos)

I. Introduction

What's the hubbub? What's the rub?
What befalls the Cosmos Club?
The molecules now stand aside
To watch the scientists collide.

Men of Math and Chemistry,
Men of Physics disagree.
Each in turn shall make his plea
To force another up a tree.

Here a man believes with vigor,
There a man contests with rigor.
Lo the skeptic! Let him try
To strike some sparks and watch them fly.

Continuum against stochastic!
Molecular against elastic!
The lines are drawn! The die is cast!
Les jeux sont faites! The words come fast.

Come sit with me. We'll take the chance
Perhaps we'll learn some great advance.
You may not wish to join my faction,
But let me give you my reaction.

II. Notes on the Addresses and Consequent Discussions

1. Subjected

Hail to thee, O Body $\mathcal{B}$!
Long live Objectivity!
Minerva-like, you sprang up whole
From out the head of Walter Noll.

O Body $\mathcal{B}$, O don't you care
If you repose or fly through air?
O Body $\mathcal{B}$, O don't you know
What speed or spin you undergo?

If on your journey, near and far,
Your German cousin $\mathfrak{K}$örper $\mathfrak{K}$
Should push you, should you know it's he?
Or could you think it might be me?

Of thee, dear $\mathcal{B}$, I must despair.
You make me tear my precious hair.
I cannot comprehend the notion
Of your indifferent constitution.

2. Confused

Concepts strange, ideas exotic,
Molecules and points chaotic.
Maxwell, Boltzmann, Grad and Ross,
Who has won and who has lost?

When to average? When to prate?
When to pause to integrate?
When mechanic? When stochastic?
When to make assumptions drastic?

How small the time? How full the box?
What the J and what the x?
Why to stop and sing this dirge?
Because the series won't converge!

3. Unstabilized

O hear the chemist mathematic!
O let him make his point emphatic!
No quasistatic change, no rates,
Just hierarchy of the states!

Gibbs and Duhem, Coleman, Noll,
Thermostatics is their goal.
Local, global, entropy,
Convex inequality.

As any fool can plainly see,
All follows from stability!
I'd like to argue, fight and goad,
But truly, sir, he's got me snowed.

4. Excited

Let us keep our self-control.
Let us hear the sage Montroll.
Hear the theorems. Hear the facts.
Hear the words of Grad and Kac.

See the Feynman diagrams;—
Revelations using plans.
Normal modes and lo,—diffusion!
Particles in gross confusion!

Messrs. Green and Lebowitz,—
Questions of the physicist.
Correlations, small omega,
Neutrons scatter, nothing vaguer.

I cannot keep my self-control!
I cannot play my passive role!
My head may burst with thoughts profuse!
But chairman Truesdell called a truce.

Poet laureate Barry Bernstein concludes a session.

5. Warped

Will open ears and eyes behold
Tensors on a manifold!
Here and there an integration,
And lo,—a law conservation!

A dire fate, a hopeless case
To find yourself in Toupin Space,
With index up and index down,
And multivectors all around.

A tensor ranked, and pranced about,
Until another summed him out.
And one was proud as proud could be
To be a tensor density.

O vectors two, O can't you tell
Whether you are parallel?
Stand still! Don't move! Don't be a drifter!
Await the judgment of the shifter!

No more distance, no more time.
Just connections all affine.
No more motions fast or slow.
World lines take you where you go.

Save me Euclid! No more! No more!
Stop this nightmare! Where's the door!?
I'd disagree! I'd joust with you!
But frankly, Dick, you've snowed me too.

III. Conclusion

The speeches done, the sun is low.
The time has come for us to go.
We leave our friends, both old and new,
With heavy hearts, with sorrow true.

With vistas new we make a note
To read the words the speakers wrote,
To learn to grasp the things they said
In sober days that lie ahead.

When next we meet we'll need not shout.
Each question will be well thought out.
But sage is he who grins and mentions
The road that's paved with good intentions.