the formulas are restricted to the particular case that $l_1 + l_2 + l_3$ (and $l_1 + j_2 + j_3$) be odd. The applicable formulas for the case when the sum is even are not given (or even referenced). (c) More extensive *algebraic* tables would have been of much value.

These flaws are minor, and, as a whole the discussion of the $(3n - j)$ symbols is very well done and includes a great deal of material in a very little space.

Mathematical Methods and Theory in Games, Programming, and Economics. By Samuel Karlin. Vol. 1, Matrix Games, Programming, and Mathematical Economics, 433 pp.; Vol. 2, The Theory of Infinite Games, 386 pp. Addison-Wesley Publishing Co., Inc., Reading, Mass., 1959. $12.50 each. *Reviewed by J. Gillis, The Weizmann Institute of Science.*

SIXTEEN years have passed since the appearance of von Neumann and Morgenstern's book inaugurated an entirely new subject for mathematical analysis. The book under review is a timely account of what it has all led up to so far. The scope of the development can be glimpsed from the bibliography at the end of each of the two volumes, while the depth attained in this subject is well presented in the book itself.

The first volume opens with four chapters devoted to the general theory of matrix games. There is a slightly unusual feature in that the emphasis is from the beginning on mixed strategies, pure strategies being regarded as a special case of no particular importance. The basic argument is always elementary but never trivial. The examples, both those solved in the text and those set as exercises, are well chosen and interesting. The reader who takes the trouble to work them all out can feel that he has grasped the subject. However, the beginner would do well to tackle first some easier and more elementary work, e.g., Kinsey, before taking up Karlin's book.

The second part of Volume 1 is devoted to linear and nonlinear programming and mathematical economics. Chapter 5 deals with the linear programming problem and presents basic existence theorems. Certain standard problems are described, warehouse problem, transportation problem, caterer's problem, and a number of others. These are not so much solved as studied analytically to make clear exactly what they involve. The following chapter is devoted to the practical solution of linear programming problems. The simplex method and its variants and the Brown algorithm constitute the bulk of the chapter. There is a discussion of the convergence of the Brown algorithm, a subject which most textbooks tactfully try to avoid. The chapter ends with a short account of the differential equations method developed by Brown and von Neumann. Chapter 7 deals with problems of nonlinear programming and all the known methods are described. In nonlinear problems special attention is given to concave programming and to duality properties.

The last two chapters of the volume deal with the mathematical analysis of economic models. It was said of Jevons and his school that in their search for a mathematical analysis of value they overlooked the value of mathematical analysis. Nobody could level this charge against Karlin's discussion. Readers unfamiliar with mathematical economics will be surprised at the depth to which some of these problems can be analyzed. Fundamental to many of the methods is the theory of positive matrices, i.e., matrices all of whose elements are positive. The investigation of such matrices began with Frobenius in 1908, but considerable work has been done on the subject in recent years, not least by the author of this book. Of economic problems we are given, among others, brilliant discussions of linear and nonlinear equilibrium models, welfare economics stability of competitive equilibrium and the von Neumann model of an expanding economy.

The first volume ends with appendixes devoted to some basic mathematics, vector spaces and matrices, convex sets and functions; and a third appendix dealing with a variety of topics in real function theory which are needed in the course of the book. The bibliography includes no fewer than 257 references.

The second volume deals with infinite games. However, as part of an effort to make the two volumes separately self-contained, Chapter 1, on finite matrix games, is a copy of Chapter 1, Volume 1, complete with examples and notes. For the same reason the appendixes and bibliography at the end of Vol. 1 are repeated at the end of the second volume.

Chapter 2 of the second volume presents a short but lucid exposition of the general idea of infinite games. The min-max theorem is proved, under suitable conditions, and the chapter ends with some eminently practical hints on how to solve infinite games. Chapter 3 is devoted to separable and polynomial games, beginning with an introduction by way of finite convex games. Most of this chapter is based on earlier work of the author and of Dresher and Shapley.

Convex and generalized convex kernels are dealt with in the fourth chapter, while Chapters 5 and 6 deal fairly thoroughly with games of timing. These have an obvious interest for those whose business it is to analyze past wars, but one may be pardoned a certain scepticism as to whether the time scale of a thermonuclear war would allow the very possibility of the considerations used here. To put it crudely, the assumptions of analyticity, or even continuity, of the functions with respect to time depend very much on the idea that the total time is large enough for small stretches of it to be written as dt.

Games of miscellaneous special types, including the author's own beautiful theory of bell-shaped games, are covered in Chapter 7, and Chapter 8 deals with infinite games "not played over the unit square". The volume ends with a chapter on poker and general parlor games.

The two volumes of this book will for some time to come be the definitive work on the subject. The exposition is lucid without verbosity and the amount of material included is impressive. The explanatory

notes and bibliographical references make the book an excellent starting point for research in the field.

A long time will pass before a realistic assessment can be made of the scientific legacy of von Neumann, since it consisted not only of new and important results, but also, and perhaps mainly, of new domains of science opened up to investigation. And there is enough in these to keep researchers busy for many years ahead. Moreover, implicit in all investigations, such as those described in Prof. Karlin's two volumes, is the idea that somewhere there is an electronic digital computer to carry out the work. And here too our debt is plain.

Introduction to Theoretical Meteorology. By Seymour L. Hess. 362 pp. Henry Holt and Co., Inc., New York, 1959. $8.50. *Reviewed by Ferguson Hall, Federal Aviation Agency.*

PROFESSOR Hess has been highly successful in meeting his stated objective of writing a book on theoretical or "dynamic" meteorology which is rigorous but not "difficult", in the sense of being bound up in advanced mathematics. Of importance also is the clarity with which each topic is presented and discussed, as well as the engaging style which makes the book readable and interesting.

The topics covered are those which underlie all atmospheric properties and motions, including solar and terrestrial radiation, the vertical state and motions of the air (convection), the thermodynamics of the atmosphere, horizontal motions (the winds), the character of fronts, the turbulent layer near the ground, and the theories of the large-scale or general circulation of the atmosphere. In addition the reader is introduced to the recent developments in "numerical weather prediction" which make use of high-speed electronic computers.

For the student of meteorology, as well as for the practicing meteorologist desiring a review of basic meteorological theory, this book seems an excellent choice. This is true also for teachers and others who wish to go a step beyond the purely descriptive aspects of weather and gain an insight into the physical laws which in concert produce each day's complex weather map.

Applications of the Theory of Matrices. By F. R. Gantmacher. Translated from Russian and revised by J. L. Brenner, D. W. Bushaw, S. Evanusa. 317 pp. Interscience Publishers, Inc., New York, 1959. $9.00. *Reviewed by Philip M. Morse, Massachusetts Institute of Technology.*

MATRICES may be of declining interest to mathematicians but they are of increasing interest to physicists, systems engineers, and operations research workers. In perturbation calculations, in programming for digital computers, in working out a Markov chain or a queuing problem, one often needs to diagonalize a matrix or compute a matrix function or solve a matrix equation. It has been pretty hard to learn these things; most of the literature on the subject is scattered and not very "solution-oriented". Consequently, the present volume, a translation of the second half of a two-volume work on matrices, is a welcome addition. In this reviewer's opinion, the first half is just as valuable. Luckily, Chelsea has now published a translation of both volumes (at a somewhat smaller price!).

The volume here reviewed includes chapters on complex symmetric, antisymmetric, and orthogonal matrices, on pencils of matrices, on matrices with nonnegative elements (important for Markov and other stochastic processes), on the application of matrix theory to the study of differential equations (important in queuing theory, for example), and on the application of matrix theory to the study of the stability of solutions of such equations (basic to control system theory). In these latter subjects the book is unique in the depth and unification of treatment of material previously nearly inaccessible. Bellman's recent volume, *Introduction to Matrix Analysis,* treats some of this material in a much more readable way, and beginners in the field are advised to start with Bellman. But if they want the real dope, they should dig through Gantmacher's Volumes 1 and 2.

Quantum Chemistry: Methods and Applications. By R. Daudel, R. Lefebvre, C. Moser. 572 pp. Interscience Publishers, Inc., New York, 1959. $14.50. *Reviewed by Stuart A. Rice, Institute for the Study of Metals, The University of Chicago.*

THERE are undoubtedly many different approaches to the teaching of quantum chemistry. Most of the books I am familiar with start with some exposition of the historical origins of quantum theory, continue with an abbreviated discussion of classical mechanics, vibrating strings, and eigenvalue problems and then with this background proceed to study the hydrogen atom and so forth. Daudel, Lefebvre, and Moser have abandoned the cited ordering of topics. Instead, they first discuss extensively empirical methods for the computation of approximate stationary states of molecules. The necessary physical concepts are introduced qualitatively and then used in the general framework of the independent electron model to treat interatomic distances; bond angles; dissociation, ionization, and resonance energies; excited states; dipole moments; as well as chemical reactivity, reaction rates, and biochemical applications. A brief discussion of the nature of the approximations in the independent electron model precedes the study of more sophisticated molecular calculations. In the second half of the book the student is introduced to the problem of electron-electron interactions as studied by a variety of methods. The hydrogen molecule and ethylene and benzene are used extensively as examples.

There is, in my opinion, a great deal to recommend in this book. In particular I find the "illogical" ordering of the subject matter very pleasing because the