

Books

The 3-*j* and 6-*j* Symbols. By Manuel Rotenberg, R. Bivins, N. Metropolis, John K. Wooten, Jr. 512 pp. The Technology Press, MIT, Cambridge, Mass., 1959. \$16.00. Reviewed by L. C. Biedenharn, *The Rice Institute*.

IN the analysis of physical problems using quantum mechanics, it is fortunate that a very large part of the relevant results may be deduced from considerations of the fundamental symmetry operations. This point of view—and its systematic development for rotational symmetry into an “angular momentum calculus”—is now standard in the literature; there exist, moreover, several excellent expositions devoted to these methods. Armed with these techniques, one can now tackle—and solve—extremely complicated problems of coupled angular momenta; the answers so found will involve the various angular momentum functions, in particular, the Racah and Wigner coefficients. Thus arises next the need for comprehensive, readily available, numerical (and algebraic) tabulations of the Racah and Wigner coefficients. It is to this need that the present volume addresses itself and, in the opinion of the reviewer, succeeds admirably.

The first question which must be answered in a prospective tabulation is what and how to tabulate. The “how” is relatively easy: the Wigner and Racah coefficients are signed square roots of rational fractions; the optimal procedure is to tabulate these rational fractions, with a special convention as to sign. The present volume utilizes a very compact scheme (devised by Sharp, Kennedy, Sears, and Hoyle) and tabulates for the squares of the angular momentum functions the exponents of the prime factors. (Such a scheme is particularly useful in applications in nuclear spectroscopy where a multiplication of several angular momentum functions is generally necessary.) “What” to tabulate is rather more difficult to answer. From the point of view of economy of space the answer is clearly in favor of Wigner’s *symmetrized* coefficients: (a) the (3*j*) symbol replaces the Wigner coefficient, i.e.,

$$\begin{pmatrix} j_1 j_2 j_3 \\ m_1 m_2 m_3 \end{pmatrix} = (2j_3 + 1)^{-1/2} (-)^{j_1 - j_2 - m_3} (j_1 m_1 j_2 m_2 | j_3 m_3).$$

(The Wigner coefficient is tailored to the addition of two angular momenta $\mathbf{a} + \mathbf{b} = \mathbf{c}$; the (3*j*) symbol is more symmetrical in that it treats the coupling of three angular momenta to form an invariant $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$.) (b) The (6*j*) symbol is a more symmetrical ver-

sion of the Racah coefficient, and differs only in sign from the original. There is probably little question in that it is best to tabulate the (3*j*) and (6*j*) symbols rather than the Wigner and Racah coefficients, even though most of the literature uses the latter.

The prime requisite of a numerical tabulation is accuracy, and the authors of this present volume have tackled this problem in an exemplary way. They have completely automated the entire process; the calculations were carried out using double precision logic (86 bits) on the Los Alamos computer MANIAC II, and the answers recorded on magnetic tapes; the tables were automatically typed in standard format, under the control of a paper tape prepared from the magnetic tape; the present book was reproduced from photographs of the output of the typewriter. To check against random machine error the entire set of tables was computed twice, and the two tapes compared, according to the authors (which seems to imply that there were no discrepancies!). The authors estimate on the basis of the single obvious *typing* error found in the entire tabulation (a missed space) that one typing error lurks somewhere in the numbers themselves. (This reviewer dutifully spot-checked a dozen or so pages of the tables quite in vain; the odds for finding an error are probably just about as stated, even including machine error.)

The tables contained in the present volume are as complete as one could reasonably desire. They include all (3*j*) and (6*j*) symbols whose arguments are ≤ 8 (proceeding, of course, in steps of $\frac{1}{2}$). In addition, the special (3*j*) symbol with all magnetic quantum numbers zero is tabulated for arguments ≤ 16 . The format of the tables is very well done; the reproduction is clear and legible. A very extensive bibliography of previous tables is included.

In addition to the numerical tables, the book contains an excellent discussion of the (3*n* - *j*) symbols by Manuel Rotenberg; this discussion is intended as introductory and is a concise—but nonetheless useful—collection of formulas relevant to the (3*j*), (6*j*), (9*j*), and even (12*j*) symbols. Short algebraic tables of the (3*j*) and (6*j*) symbols are included. This material is very well treated; it is thoroughly referenced and very much up to date.

It is possible to point out some minor flaws in this introductory material (to conform to the unwritten rule for reviewing!), and for what is worth here are some: (a) Although a complete compendium of notations for the various coefficients might be expected, this is only partially done. In fact, one of the footnotes (p. 5) uses an undefined notation. (b) In giving the relation of the symbols

$$\begin{pmatrix} l_1 l_2 l_3 \\ 0 \ 1 \ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} l_1 j_2 j_3 \\ 0 \ \frac{1}{2} \ -\frac{1}{2} \end{pmatrix}$$

to the specially tabulated symbol

$$\begin{pmatrix} l_1 l_2 l_3 \\ 0 \ 0 \ 0 \end{pmatrix}$$

WILEY**BOOKS****FUNDAMENTAL PHYSICS**

By JAY OREAR, *Cornell University*. Emphasizes understanding of basic principles and their relation to all of physical science. January, 1961. Approx. 456 pages. Prob. \$7.00.

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By H. C. CORBEN, *Ramo-Wooldridge Research Laboratory*, and PHILIP STEHLE, *University of Pittsburgh*. Stresses both abstract theory and practical application. 1960. Approx. 408 pages. Prob. \$10.00.

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Including the Proceedings of the Conference on Thermo-electricity Sponsored by the Naval Research Laboratory

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By WARD C. SANGREN, *General Dynamics Corporation*. Helps to acquaint nuclear engineers and scientists with the use of digital computers in the design of nuclear reactors. 1960. 208 pages. \$8.50.

CONFERENCE on VERY HIGH PRESSURE

Edited by F. BUNDY, W. R. HIBBARD, and H. M. STRONG, *all of the General Electric Research Laboratory, Schenectady, N. Y.* In Press.

THEORY of NEUTRAL and IONIZED GASES

A Series of Lectures given at the 1959 Summer Session for Theoretical Physics at Les Houches

Edited by CÉCILE DE WITT, *Director of Les Houches*; and JEAN-FRANÇOIS DETOEUF, *Associate Director*. With 7 contributors. 1960. 469 pages. \$17.50.

PROPERTIES and STRUCTURE of POLYMERS

By ARTHUR V. TOBOLSKY, *Princeton University*. 1960. 331 pages. \$14.50.

INTRODUCTION to MODERN NETWORK SYNTHESIS

By M. E. VAN VALKENBURG, *University of Illinois*. 1960. 498 pages. \$11.75.*

X-RAY ABSORPTION and EMISSION in ANALYTICAL CHEMISTRY

By H. A. LIEBHAFSKY, H. G. PFEIFFER, E. H. WINSLOW, and P. D. ZEMANY, *all of the General Electric Research Laboratory, Schenectady, N. Y.* 1960. 357 pages. \$13.50.

COMPARATIVE EFFECTS of RADIATION

Edited by MILTON BURTON, *University of Notre Dame*; JOHN S. KIRBY-SMITH, *Oak Ridge National Laboratory*; and JOHN L. MAGEE, *University of Notre Dame*. 1960. In Press.

The DYNAMIC BEHAVIOR of THERMOELECTRIC DEVICES

By PAUL E. GRAY, *Massachusetts Institute of Technology*. A Technology Press Research Monograph. 1960. 136 pages. \$3.50.

The SURFACE CHEMISTRY of METALS and SEMICONDUCTORS

Edited by HARRY C. GATOS, *M.I.T.*, with the assistance of J. W. FAUST, and W. J. LAFLEUR. 1960. 526 pages. \$12.50.

PRINCIPLES of SEMICONDUCTOR DEVICE OPERATION

By A. K. JONSCHER, *Research Laboratories of General Electric Co., Ltd., England*. 1960. In Press.

PROCEEDINGS of the SECOND CONFERENCE on REACTIONS BETWEEN COMPLEX NUCLEI

Held in Gatlinburg, Tenn., May 2-4, 1960.

Edited by ALEXANDER ZUCKER, EDITH C. HALBERT, and FREDERICK T. HOWARD, *all of the Oak Ridge National Laboratory*. 1960. 319 pages. \$7.00.

ELECTRONIC PROCESSES in SOLIDS

By PIERRE R. AIGRAIN, *Université de Paris*, ROLAND J. COELHO, and GIANNI ASCARELLI. A Technology Press Book, M.I.T. 1960. 67 pages. \$4.00.

CASE STUDIES in ELECTROMAGNETISM

By HERMANN A. HAUS and JOHN P. PENHUNE, *both of M.I.T.* 1960. 335 pages. \$6.50.

SEMICONDUCTOR ABSTRACTS, Volume VI, 1958 Issue

Edited by J. J. BULLOFF and C. S. PEET, *Battelle Memorial Institute*. 1960. In Press.

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JOHN WILEY & SONS, Inc.**440 Park Avenue South, New York 16, N.Y.**

OCTOBER 1960

the formulas are restricted to the particular case that $l_1 + l_2 + l_3$ (and $l_1 + j_2 + j_3$) be odd. The applicable formulas for the case when the sum is even are not given (or even referenced). (c) More extensive algebraic tables would have been of much value.

These flaws are minor, and, as a whole the discussion of the $(3n - j)$ symbols is very well done and includes a great deal of material in a very little space.

Mathematical Methods and Theory in Games, Programming, and Economics. By Samuel Karlin. Vol. 1, Matrix Games, Programming, and Mathematical Economics, 433 pp.; Vol. 2, The Theory of Infinite Games, 386 pp. Addison-Wesley Publishing Co., Inc., Reading, Mass., 1959. \$12.50 each. Reviewed by J. Gillis, *The Weizmann Institute of Science*.

SIXTEEN years have passed since the appearance of von Neumann and Morgenstern's book inaugurated an entirely new subject for mathematical analysis. The book under review is a timely account of what it has all led up to so far. The scope of the development can be glimpsed from the bibliography at the end of each of the two volumes, while the depth attained in this subject is well presented in the book itself.

The first volume opens with four chapters devoted to the general theory of matrix games. There is a slightly unusual feature in that the emphasis is from the beginning on mixed strategies, pure strategies being regarded as a special case of no particular importance. The basic argument is always elementary but never trivial. The examples, both those solved in the text and those set as exercises, are well chosen and interesting. The reader who takes the trouble to work them all out can feel that he has grasped the subject. However, the beginner would do well to tackle first some easier and more elementary work, e.g., Kinsey, before taking up Karlin's book.

The second part of Volume 1 is devoted to linear and nonlinear programming and mathematical economics. Chapter 5 deals with the linear programming problem and presents basic existence theorems. Certain standard problems are described, warehouse problem, transportation problem, caterer's problem, and a number of others. These are not so much solved as studied analytically to make clear exactly what they involve. The following chapter is devoted to the practical solution of linear programming problems. The simplex method and its variants and the Brown algorithm constitute the bulk of the chapter. There is a discussion of the convergence of the Brown algorithm, a subject which most textbooks tactfully try to avoid. The chapter ends with a short account of the differential equations method developed by Brown and von Neumann. Chapter 7 deals with problems of nonlinear programming and all the known methods are described. In nonlinear problems special attention is given to concave programming and to duality properties.

The last two chapters of the volume deal with the mathematical analysis of economic models. It was said

of Jevons and his school that in their search for a mathematical analysis of value they overlooked the value of mathematical analysis. Nobody could level this charge against Karlin's discussion. Readers unfamiliar with mathematical economics will be surprised at the depth to which some of these problems can be analyzed. Fundamental to many of the methods is the theory of positive matrices, i.e., matrices all of whose elements are positive. The investigation of such matrices began with Frobenius in 1908, but considerable work has been done on the subject in recent years, not least by the author of this book. Of economic problems we are given, among others, brilliant discussions of linear and nonlinear equilibrium models, welfare economics stability of competitive equilibrium and the von Neumann model of an expanding economy.

The first volume ends with appendixes devoted to some basic mathematics, vector spaces and matrices, convex sets and functions; and a third appendix dealing with a variety of topics in real function theory which are needed in the course of the book. The bibliography includes no fewer than 257 references.

The second volume deals with infinite games. However, as part of an effort to make the two volumes separately self-contained, Chapter 1, on finite matrix games, is a copy of Chapter 1, Volume 1, complete with examples and notes. For the same reason the appendixes and bibliography at the end of Vol. 1 are repeated at the end of the second volume.

Chapter 2 of the second volume presents a short but lucid exposition of the general idea of infinite games. The min-max theorem is proved, under suitable conditions, and the chapter ends with some eminently practical hints on how to solve infinite games. Chapter 3 is devoted to separable and polynomial games, beginning with an introduction by way of finite convex games. Most of this chapter is based on earlier work of the author and of Dresher and Shapley.

Convex and generalized convex kernels are dealt with in the fourth chapter, while Chapters 5 and 6 deal fairly thoroughly with games of timing. These have an obvious interest for those whose business it is to analyze past wars, but one may be pardoned a certain scepticism as to whether the time scale of a thermonuclear war would allow the very possibility of the considerations used here. To put it crudely, the assumptions of analyticity, or even continuity, of the functions with respect to time depend very much on the idea that the total time is large enough for small stretches of it to be written as dt .

Games of miscellaneous special types, including the author's own beautiful theory of bell-shaped games, are covered in Chapter 7, and Chapter 8 deals with infinite games "not played over the unit square". The volume ends with a chapter on poker and general parlor games.

The two volumes of this book will for some time to come be the definitive work on the subject. The exposition is lucid without verbosity and the amount of material included is impressive. The explanatory