

Books

An Introduction to Fourier Analysis and Generalised Functions. By M. J. Lighthill. 79 pp. Cambridge U. Press, New York, 1958. \$3.50. *Reviewed by Freeman J. Dyson, Institute for Advanced Study.*

The theory of generalized functions (alias distributions) is the only part of post-war pure mathematics which has turned out to have a genuine usefulness in physics. In fashionable field-theoretic circles a paper can now hardly be submitted for publication without at least a reference to Schwartz's two-volume *Théorie des Distributions*. In some recent papers one can find evidence that Schwartz's work has not only been quoted but has even been read. Applications of distribution theory are being found all over physics, in nonlinear mechanics and fluid dynamics as well as in field theory.

Distribution theory is a new style of function theory, in which all the dubious tricks to which physicists are prone (Dirac delta-functions, differentiation of non-differentiable functions, and especially the free use of diverging Fourier series and integrals) are simply and rigorously justified. The original theory of Schwartz has been boiled down and predigested for the use of physicists by Temple (papers in *Proc. Roy. Soc.*, 1955 and 1956). Lighthill has now taken the logical last step; this book presents the essential ideas of distribution theory in so clear and self-contained a form that they can be offered directly to undergraduates. The book is in fact a first course in Fourier analysis, given to final-year undergraduates at Manchester University, making use of distribution theory from the beginning and so avoiding the whole huge detour of the classical treatment of trigonometric series. The dedication is "To Paul Dirac, who saw that it must be true, Laurent Schwartz, who proved it, and George Temple, who saw how simple it could be made".

The qualifications of Lighthill to write such a book are unique. In the first place, he is a practicing aerodynamicist; he is not afraid to get his hands dirty and let formal elegance be damned. In the second place, he sat for two years at the lectures of G. H. Hardy, the greatest stylist of his era in pure mathematics, and the personification of the classical tradition of Fourier analysis. Hardy's lectures, at which Lighthill and I and two other students were the entire audience, were always prepared with impeccable care and delicacy, and delivered with humor and informality. The subject matter was a sustained exercise of mathematical virtuosity, polished until it had the contrived spontaneity of a violinist's carefully rehearsed cadenza.

The highest praise one can offer to Lighthill's book is to say that it lays Hardy's work in ruins, and that Hardy would have enjoyed it more than anybody. It is written in a language that undergraduates (at least in England) can understand, with discussion of examples that physicists can use, and yet there is not the slightest departure from strict mathematical rigor. It shows conclusively what physicists have long believed, that the deep problems of convergence and uniqueness to which the classical theory of Fourier series was addressed are irrelevant for all practical purposes. Hardy himself once wrote "I count Maxwell and Einstein, Edington and Dirac, among real mathematicians." If he were alive, he would rejoice to see his student so triumphantly prove him right.

It is to be hoped that this book will quickly be adopted as the appropriate introduction to Fourier analysis for students both in physics and in mathematics. It is likely also to have a long life as a work of reference for research scientists. The table on page 43, of Fourier transforms of powers and logarithms, will soon be the most heavily thumbed in my library. My only regret is that Lighthill decided to confine himself to the one-dimensional theory. This was undoubtedly the right decision for his undergraduate audience, and made it possible to cover the whole field in 80 small pages. For the working physicist, a longer book including the standard singular functions in 2, 3, and 4 dimensions would have been even better. Perhaps Lighthill could be persuaded to write it.

White Dwarfs. By E. Schatzman. 180 pp. (North-Holland, Holland) Interscience Publishers, Inc., New York, 1958. \$5.50. *Reviewed by C. C. Kiess, National Bureau of Standards.*

The white dwarfs are a class of stars, about 200 in number, that are of very low luminosity, of small volume, and probably of small mass, but of extraordinarily high density. These qualities, derived from trustworthy though limited data of observation, indicate that the "white dwarfs form a group of stars, distinct from the others, whose origin is still unknown". Spectrographic investigations, necessarily of low dispersion, reveal for them either a featureless continuous spectrum, or a spectrum in which appear broad, shallow absorption lines of hydrogen, or of helium, or of ionized calcium. Under certain conditions emission lines of hydrogen may appear also. The enormous densities of these stars can be accounted for only by the existence, in their interiors, of matter in a degenerate state, consisting of compacted atomic nuclei that have lost nearly all of their extranuclear electrons.

Following these introductory ideas, presented in the first two chapters, the author devotes five chapters to the mathematical analysis of the problems presented by the spectra of thin gaseous strata overlying much denser substrata. Model systems with assumed parametral properties are set up and then from the analyses are derived the conditions for thermal equilibrium and

dynamical stability necessary to produce the observed characteristics of these stars. Essential to any successful model is a satisfactory mechanism for generating the internal energy that escapes from the stars; for "in view of the high density and low luminosity very strict conditions have to be satisfied". In the final chapter are offered speculations on the origin and evolution of the white dwarfs—speculations that suggest the problems of future research rather than crystallize our ideas around any of the current hypotheses.

This book is obviously written for students of astrophysics who wish to know the role of modern physics in interpreting a class of stars which, thus far, has stood apart from the main-sequence stars. To read the book with understanding and appreciation the student must have a knowledge of atomic physics and a fair acquaintance with astrophysics. To aid him in further study of any of the problems discussed in the book he will find ample citations to original papers in the bibliographical references appended to each chapter. Owing to the slow rate at which observational data on the white dwarfs accumulates it probably will be a long time before this book is supplanted.

Théorie Synthétique de la Relativité Restreinte et des Quanta. By O. Costa de Beauregard. 200 pp. Gauthier-Villars, Paris, France, 1957. Paperbound \$9.30. Reviewed by Manuel Sandoval Vallarta, University of Mexico.

The author is a distinguished member of Louis de Broglie's school of theoretical physicists. He belongs to that select—and altogether rare—group of physicists who demand consistency and a good measure of rigor in their work. Nowhere perhaps in the whole realm of theoretical physics is this approach more required and more appreciated than in the chapters which deal with the union of relativity and quantum field theory. The need for a satisfactory approach, emphasized by Tomonaga during the war years, is the main point of the book under review.

The reader will look in vain for such topics of paramount importance as a treatment of infinities, renormalization, or the self-consistency of quantum electrode names. Nor will he find calculations of effects arising essentially from field quantization, such as the theory of the Lamb shift or the anomalous magnetic moment of the electron. Nothing is said about the two-body equation of Bethe, Salpeter, Nambu, and Schwinger. No emphasis is given to the group-theoretical approach which has proved so fruitful in many instances in the past and which should be at the very core of any attempt to place the central problem dealt with in this book on a rigorous basis.

The main contribution of the author is the relativistic theory of reciprocal Fourier integrals, starting from the work of Marcel Riesz. A searching analysis is made of complementarity in Bohr's sense and of the paradoxes brought about by the probability interpretation of wave mechanics, underlined by the controversy between Ein-



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