The new math of network optimization, courtesy of string theory

Modeling the shapes of tree branches, neurons, and blood vessels is a thorny problem, but researchers have just discovered that much of the math has already been done.

By Johanna L. Miller

Social networks, neural networks, computer networks: So many of the things we mean when we talk about networks are abstract or virtual structures, characterized by the mathematical connections among nodes rather than their physical arrangement in space. When physicists have joined the interdisciplinary field of network science, those are the types of systems they've tended to focus on. But in the past few years, some have turned their attention to physical networks, such as real neurons (depicted in figure 1) and blood vessels, and researchers have asked, Why do those structures have the shapes that they do?

Some possible answers can be ruled out. For example, one might guess that networks grow in a way that minimizes the total length of the links needed to connect a set of points in space. But it can be proved that, if that were true, network links would always join at

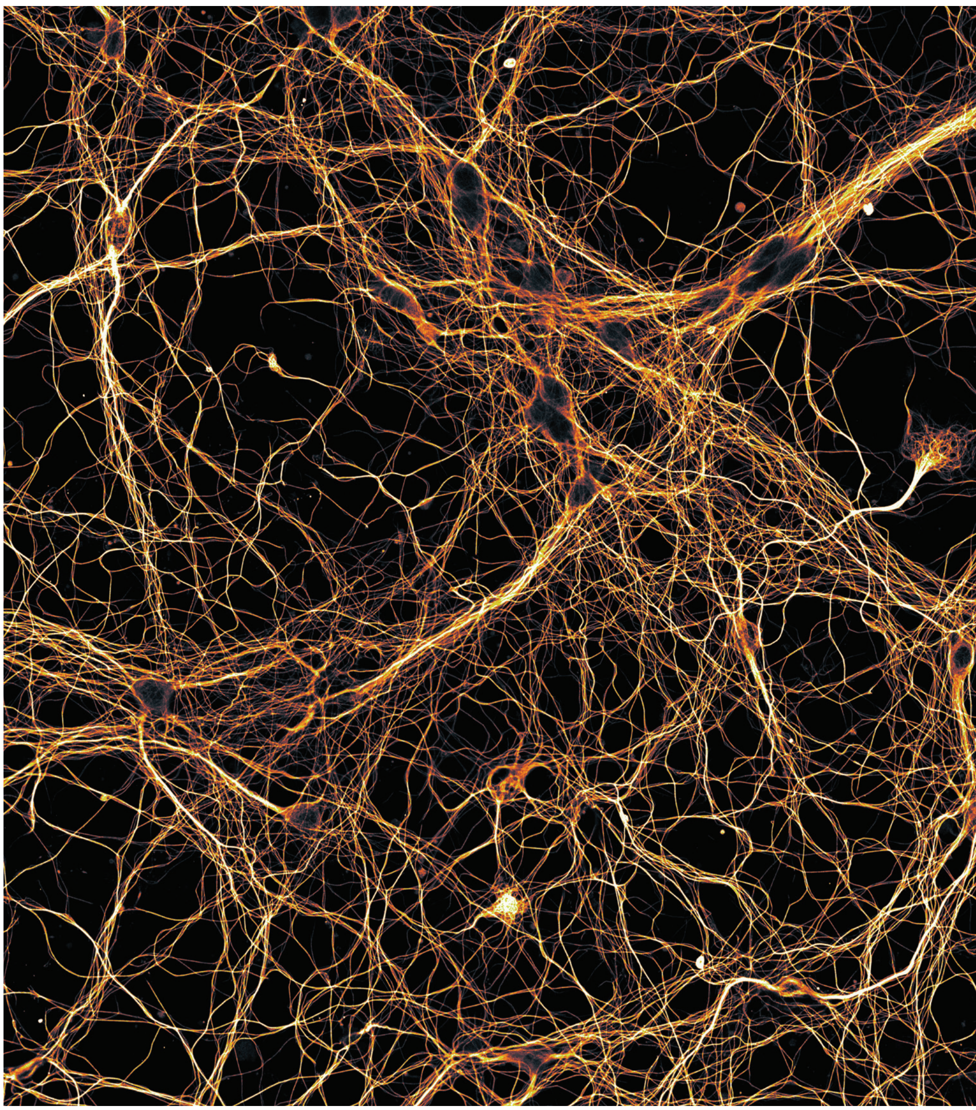

Figure 1. Real neurons, such as the ones shown here from a rat's hippocampus, are an example of a physical network whose structure is constrained by material resources. Examining the constraints can lead to insights into network geometry. (Image by MikeRoscopy/Wikimedia Commons/CC BY 4.0.)

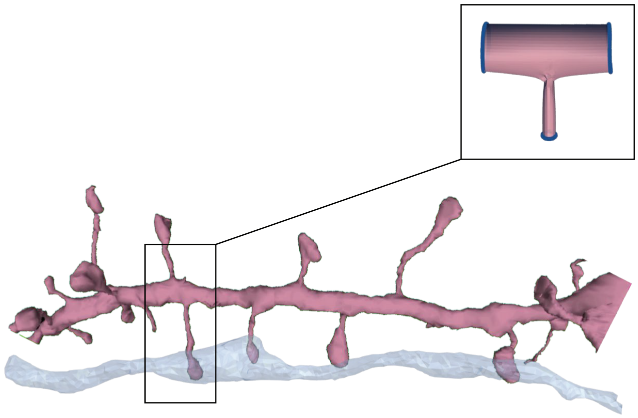

Figure 2. Orthogonal sprouts—thin network branches that emerge from a much thicker straight spine—are predicted by the surface-minimization hypothesis, and they appear in many real physical networks. (Image adapted from ref. 1.)

planar, 120° junctions—and they don't. Now Albert-László Barabási, of the Network Science Institute at Northeastern University in Boston, and colleagues have found that physical network structures can be better explained by a different hypothesis: The networks seek to minimize not their length but their surface area.[1]

It's easy to see why surface area should be the most relevant quantity for networks of blood vessels and other tubelike structures that are nothing but surface area. For other systems, such as tree branches and nerve cells, the outer surface of a network link is often the most biologically expensive to construct. In testing the hypothesis, however, the researchers hit a wall: Calculating the surface-area-minimizing network structure is a computationally intractable problem. There are too many possibilities for the thickness and position of all the network branches, and it's too difficult to exactly model the junctions where branches smoothly join together.

But then Xiangyi Meng—Barabási's collaborator and former postdoc, now on the faculty at Rensselaer Polytechnic Institute—made a critical discovery: String theory, he realized, had already grappled with a mathematically equivalent problem. In string theory, reactions among elementary particles are represented as continuous branched manifolds called worldsheets, which evolve in a way that minimizes their surface area. Exactly solving the surface-minimization problem is just as intractable for string theorists as it is for network theorists. But the string theorists had a decades-long head start, and they developed mathematical tools to find useful approximate solutions.

By piggybacking on the string-theory solution, Barabási and colleagues calculated what the surface-minimizing physical networks should look like, and the features they found agreed well with observations. For example, they found that networks under a wide variety of conditions should contain the structures in figure 2, which they call orthogonal sprouts: thin tubes that emerge at a 90° angle from a straight, much wider tube. Orthogonal sprouts don't show up in most other network models. But they appear in many real-life networks, including corals, fungi, blood vessels, and tree roots. PT

Reference

1. X. Meng et al., "Surface optimization governs the local design of physical networks," *Nature* **649**, 315 (2026).